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A

KEY

89

TO

HUTTON'S COMPENDIOUS MEASURER;

CONTAINING

SOLUTIONS, AT FULL LENGTH,

TO ALL THE

QUESTIONS

WHICH HAVE ONLY THE ANSWERS ANNEXED TO THEM
IN THAT WORK.

BY J. M. EDNEY,

TEACHER OF THE MATHEMATICS.

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TO

OLINTHUS GREGORY, LL.D.

ASSOCIATE ACAD. DIJON, HONORARY MEMBER OF THE LITERARY
AND PHILOSOPHICAL SOCIETY OF NEW YORK;
MEMBER OF THE ASTRONOMICAL SOCIETY OF LONDON,
&c. &c. &c. AND PROFESSOR OF
MATHEMATICS IN THE ROYAL MILITARY
ACADEMY, WOOLWICH,

THIS WORK

IS

RESPECTFULLY DEDICATED

BY

HIS OBLIGED AND FAITHFUL SERVANT,

J. M. EDNEY.

PREFACE.

Dr. Hutton's Compendious Measurer being a work of high and established reputation, no apology can be necessary for an attempt to make it more generally useful, both to the tutor and the learner, by supplying the solution to every unanswered question in that work, in the form of a key.

In order to accomplish this object, endeavours have been used to work the questions in the easiest and plainest manner, and, as far as possible, in conformity with the rules laid down in that work: at the same time, care has been taken to unite perspicuity with conciseness. If the author has succeeded so far as to render the treatise more serviceable, and thereby to facilitate the acquisition of useful science, the purpose he had in view will be fully answered.

The student will observe, that, in a few instances, the answer inserted in the Key does not exactly correspond with that in the original work; but where this is the case, the error, it will be found, is not in the Key, but has been overlooked in the treatise itself.

St. John-Street, Clerkenwell,
January, 1824.

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K E Y

TO

HUTTON'S COMPENDIOUS MEASURER.

ADDITION OF DECIMALS. (Page 4 *).

Ex. 4. 72914·47944. Ex. 5. 309488·2918.
Ex. 6. 3646·2088.

SUBTRACTION. (Page 5).

Ex. 4. 315·287. Ex. 5. 2713·084. Ex. 6. 783765.

MULTIPLICATION. (Page 6).

Ex. 4. 1083·6. Ex. 5. 9·106. Ex. 6. ·0004641.

CASE II.

Ex. 2. 271·4. Ex. 3. 916·3. Ex. 4. 218100.

---

* The page refers to the situation of the problem or question in the Treatise.

The contracted Method. (Page 7).

Ex: 2.

$$\begin{array}{r}
480\cdot14936 \\
61427\cdot2 \\
\hline
9602987 \\
3361045 \\
96030 \\
19206 \\
480 \\
288 \\
\hline
1308\cdot0036 \\
\hline
\end{array}$$

Ex. 3.

$$\begin{array}{r}
2490\cdot3048 \\
68\cdot2375 \\
\hline
124515240 \\
17432134 \\
747091 \\
49806 \\
19922 \\
1494 \\
\hline
1427\cdot65687 \\
\hline
\end{array}$$

Ex. 4.

$$\begin{array}{r}
325\cdot701428 \\
38127\cdot \\
\hline
227990 \\
6514 \\
326 \\
260 \\
10 \\
\hline
235\cdot100 \\
\hline
\end{array}$$

DIVISION. (Page 9).

Ex. 3. $\cdot0236$. Ex. 4. $39\cdot6111\dot{1}$ Ex. 5. $\cdot0084$.
Ex. 6. $506\cdot9767$.

Contractions. (Page 9).

Ex. 3. Here $\dfrac{45\cdot5}{2170}=\dfrac{4\cdot55}{217}=\cdot02096$.

Ex. 4. Here $\dfrac{61}{79000}=\dfrac{\cdot061}{79}=\cdot0007721$.

CASE II.

$$\frac{5\cdot16}{1000} = \cdot00516 \quad \frac{419}{10} = 41\cdot9 \quad \frac{\cdot21}{1000} = \cdot00021.$$

CASE III. (Page 10).

Ex. 2. In this instance the quotient will contain six figures; viz. two whole numbers (rule 4, page 8), and four decimals, as stated in the question.

```
230·409)4109·23,51(17·8345
             180514
              19228
                795
                104
                 12
```

```
Ex. 3.   5713·9,6)37·1043,8(·0064̇937
                   28209
                    5354
                     212
                      41
                       1
```

```
Ex. 4.   213,7·2)913,08(·428
                 61
                 18
                 1
```

REDUCTION OF DECIMALS. (Page 11).

$\frac{2}{7} = \cdot\dot{2}8517\dot{4}$ $\quad \frac{14}{191} = \cdot732984$

$\frac{1}{758} = \cdot0013192.$

(Page 14).

Ex. 1.	£.	Ex. 2.	s.	Ex. 3.	£.
	·775		·625		·8635
	20		12		20
	15·500		7·500		17·2700
	12		4		12
	6·000		2·000		3·2400
	15*s.* 6*d.*		7½*d.*		17*s.* 3*d.*

lb.
Ex. 4. ·0125
12
·1500
20
3·0000
3 dwts.

lb.
Ex. 5. ·4694
12
5·6328
20
12·6560
24
26240
13120
15·7440
5 oz. 12 dwts. 15 gr.

cwt.
Ex. 6. ·625
4
2·500
28
14·000
2 qr. 14 lbs.

mile.
Ex. 7. ·009943
1760
596580
69601
9943
17·499680
3
1·499040
12
5·988480
17 yds. 1 f. 6 in. nearly.

yard cloth.
Ex. 8. ·6875
4
2·7500
4
3·0000
2 qu. 3 nls.

acre.
Ex. 9. ·3375
4
1·3500
40
14·0000
1 r. 14 pls.

Ex. 10. ·2083
63
6249
12498
13·1229
13 gallons.

qr. corn.
Ex. 11. ·40625
8
3·25000
4
1·00000
3 bu. 1 p.

month.
Ex. 12. ·42857
4

1·71428
7

4·99996

1 w. 5 days, nearly.

(Page 15).

d.
Ex. 3. 12)·26

20)·0216666

·00108$\dot{3}$*l.*

Ex. 4. 16)7 drams

16) ·4375

·02734375 lb.

Ex. 5. lb.
112)2·15(·019196 cwt.
1030
720
48

Ex. 6. yards.
1760)24·00(·0136$\dot{3}\dot{6}$ mile
6400
11200
640

pole.
Ex. 7. 40)·056

4)·0014

·00035 acre

pint.
Ex. 8. 8)1·20

63) ·150(·00238 hhd.
240
510
6

min.
Ex. 9. 60)14·000

24) ·2333

·00972$\dot{2}$ day.

pint.
Ex. 10. 16)·21

·013125 peck

(Page 16.)

Ex. 3. 4)1·0
12)3·25
20)17·27083$\dot{3}$
19·863541$\dot{6}$*l.*

Ex. 4. 12)6·0
20)15·5
·775*l.*

Ex. 5. 4)2·0
12)7·5
·625*s.*

Ex. 6. 28)14·
4) 2·5
3·625 cwt.

Ex. 7. 12)6
3)1·5
7600
5600
3200
14400
320

Ex. 8. 4)3
4)2·75
·6875 yd.

Ex. 9. 40)14
4)1·35
13·3375 acre.

Ex. 10. 8)1·
63)13·125(·2083$\dot{3}$ hhd.
525
210
210
21

Ex. 11. 4)1·
8)3·25
·40625 qr.

```
Ex. 12.  7)5·0
           ―――――
          4)1·71428
            ―――――
            3·42857 month.
            ―――――
```

SQUARE ROOT. (Page 34).

```
Ex. 7.  6̇(2·449490
        4
      ―――
    44)200
        176
      ―――
    484)2400
         1936
        ―――
    4889)46400
          44001
        ――――
    48984)239900
            195936
          ――――
    489889)4396400
             4408641 nearly
           ――――――
```

```
Ex. 8.  7̇(2·645751
        4
      ―――
    46)300
        276
      ―――
    524)2400
         2096
        ―――
    5285)30400
          26425
        ――――
    52907)397500
            370349
          ――――
    529145)2715100
             2645725
           ――――――
    5291501)6937500
              5291501
            ――――――
              1645999
            ――――――
```

Ex. 9. $1\dot{0}$(3·162277
9
61)100
61
626)3900
3756
6322)14400
12644
63242)175600
126484
632447)4911600
4447129
6324547)46447100
44271829
2175271

Ex. 10. $1\dot{1}$(3·31662
9
63)200
189
661)1100
661
6626)43900
39756
66326)414400
397956
663322)1644400
1326644
6633244)31775600
26532976
5242624

BIQUADRATE, OR FOURTH ROOT. (Page 37).

Ex. 2. $\dot{2}1\dot{0}3\dot{5}\cdot 8$(145·037235158760
1

24)110
96

285)1435
1425

29003)108000
87009

290067)2099100
2030469

2900742) 6863100
5801484

29007443)106161600
87022329

290074465)1913927100
1450372325

290074470) 463554775(15876
170480305
25443070
2205958
175637
1593

```
145·037235158760(12·04314 = root required.
 1
 ___
 22)45
    44
    ______
2404)10372
      9616
      ______
24083) 75635
       72249
       ______
240861) 338615
        240861
        _______
2408624) 9775487
         9634496
         _______
24086280) 140991
          _______
```

Ex. 3. By example 4, page 33 of the Treatise, the square root of 2 is found to be as follows:

1·414213562373.

```
Hence  1·414213562373(1·189207 = root required.
       1
       ___
  21) 41
      21
      ____
  228)2042
      1824
      _____
  2369)21813
       21321
       ______
  23782) 49256
         47564
         ________
  2378407)16922373
          16648849
          ________
            273524
          ________
```

CUBE ROOT. (Page 40).

Note.—For another rule for the extraction of the cube root, which is in more general practice, see Crosby's edition of Walkingame's Tutor's Assistant.

Ex. 1. $38\dot{9}01\dot{7}$(73 = root required.
343

46017 resolvend

147
21

1491 divisor

441
189
27

46017 subtrahend.

......

Ex. 2. $\dot{1}09\dot{2}72\dot{7}$(103 = root required.
1

92727 resolvend

3
3

33 1st divisor

300
30

3030 2nd divisor

900
270
27

92727 subtrahend.

......

```
Ex. 3.  27054036008(3002 root required.
        27
       ----------------
           54036008 resolvend
          -------------
          27
            9
          ----
          279  = 1st divisor
          ----
          2700
             90
         ------
         27090 = 2nd divisor
         ------
         270000
            900
         -------
         2700900 = 3rd divisor
         -------
          540000
             3600
                8
          --------
          54036008 subtrahend.
          --------
          ........
          --------
```

Ex. 4. 219365327791(6031 = root required.
216

3365327 = 1st resolvend

108
18

1098 = 1st divisor

10800
180

108180 = 2nd divisor

32400
1620
27

3256227 = 1st subtrahend

109100791 = 2nd resolvend

1090827
1809

10910079 = 3rd divisor

1090827
1809
1

109100791 = 2nd subtrahend.

. . . .

Ex. 5. $12\dot{2}61\dot{5}32\dot{7}23\dot{2}$(4968 = root required.
64
58615 = 1st resolvend
48
12
492 = 1st divisor
432
972
729
53649 = 1st subtrahend
4966327 = 2nd resolvend
7203
147
72177 = 2nd divisor
43218
5292
216
4374936 = 2nd subtrahend
591391232 = 3rd resolvend
738048
1488
7381968 = 3rd divisor
5904384
95232
512
591391232 = 3rd subtrahend.
. . . .

To extract any Root whatever. (Page 44).

Ex. 1. Here, by trials, the nearest root is found to be 1·2.

Hence $G = 2$. $n = 3$. $A = 1{\cdot}728$, and $r = 1{\cdot}2$,

$$\overline{n+1} \times A = 4 \times 1{\cdot}728 = 6{\cdot}912$$
$$\overline{n-1} \times G = 2 \times 2 = 4$$
$$\overline{10{\cdot}912}$$

Again, $\overline{n+1} \times G = 4 \times 2 = 8$

$$\overline{n-1} \times A = 2 \times 1{\cdot}428 = 3{\cdot}456$$
$$\overline{11{\cdot}456}$$

As $10{\cdot}912 : 11{\cdot}456 :: 1{\cdot}2$

$1{\cdot}2$

10·912)13·7472(1·25
28352
65280
107200

Second Operation.

Here $G = 2$. $n = 3$. $A = 1{\cdot}953125$. $r = 1{\cdot}25$,

$$\overline{n+1} \times A = 4 \times 1{\cdot}953125 = 7{\cdot}8125$$
$$\overline{n-1} \times G = 2 \times 2 = 4$$
$$\overline{11{\cdot}8125}$$

Again, $\overline{n+1} \times G = 4 \times 2 = 8$

$$\overline{n-1} \times A = 2 \times 1{\cdot}953125 = 3{\cdot}90625$$
$$\overline{11{\cdot}90625}$$

As 11·8125 : 11·90625 :: 1·25
125

11·8125)1·48828125(1·259921 nearly,
307031
707812
1171875
1087500
243750
7500

Consequently the root required is 1·259921.

Ex. 2. The root is found to be between 1·1 and 1·2. Taking 1·1, its fourth power is 1·4641.

Hence $G = 2$. $n = 4$, $r = 1\cdot1$, $A = 1\cdot4641$,

$$\overline{n+1} \times A = 5 \times 1\cdot4641 = 7\cdot3205$$
$$\overline{n-1} \times G = 3 \times 2 = 6$$

13·3205

$$\overline{n+1} \times G = 5 \times 2 = 10$$
$$\overline{n-1} \times A = 3 \times 1\cdot4641 = 4\cdot3923$$

14·3923

As 13·3205 : 14·3923 : : 1·1
1·1

13·3205)15·83153(1·18
251103
1178980
113341

Second. Operation.

Here $G = 2$, $n = 4$, $r = 1\cdot18$, $A = 1\cdot9387776$,

$$\overline{n+1} \times A = 5 \times 1\cdot9387776 = 9\cdot6938888$$
$$\overline{n-1} \times G = 3 \times 2 = 6$$

15·6938888

$$\overline{n+1} \times G = 5 \times 2 = 10$$
$$\overline{n-1} \times A = 3 \times 1\cdot9387776 = 5\cdot81633328$$

15·81633328

As 15·6938888 : 15·81633328 :: 1·18
1·18

12653066624
17397966608

15·6938888)18·6632732704(1·18920
296938447
1399995590
1444844864
323948720
10070944

Consequently the root required is 1·18920.

Ex. 3. The root is found to be between 3·1 and 3·2. Taking 3·1, its fourth power is 92·3521.

Hence G = 97·41, n = 4, r = 3·1, A = 92·3521,

$\overline{n+1} \times \text{A} = 5 \times 92{\cdot}3521 = 461{\cdot}7605$
$\overline{n-1} \times \text{G} = 3 \times 97{\cdot}41 = 292{\cdot}23$

753·9905

$\overline{n+1} \times \text{G} = 5 \times 97{\cdot}41 = 487{\cdot}05$
$\overline{n-1} \times \text{A} = 3 \times 92{\cdot}3251 = 276{\cdot}9753$

764·0253

As 753·9905 : 764·0253 :: 3·1
3·1

7640253
22920759

753·9905)2368·47843(3·14
10650693
3110788 0
948260

Second Operation.

Here $\text{G} = 97{\cdot}41$, $n = 4$, $r = 3{\cdot}14$, $\text{A} = 97{\cdot}21171216$,

$$\begin{array}{lr} \overline{n+1} \times \text{A} = 5 \times 97{\cdot}21171216 = & 486{\cdot}05856080 \\ \overline{n-1} \times \text{G} = 3 \times 97{\cdot}41 = & 292{\cdot}23 \\ \hline & 778{\cdot}2885608 \end{array}$$

$$\begin{array}{lr} \overline{n+1} \times \text{G} = 5 \times 97{\cdot}41 = & 487{\cdot}05 \\ \overline{n-1} \times \text{A} = 3 \times 97{\cdot}21171216 = & 291{\cdot}63513648 \\ \hline & 778{\cdot}68513648 \end{array}$$

As 778·2885608 : 778·68513648 : : 3·14

$$\begin{array}{r} 3{\cdot}14 \\ \hline 311474054592 \\ 77868513648 \\ 233605540944 \\ \hline \end{array}$$

778·2885608)2445·0713285472(3·141599
11020564614
32376790067
12452476352
46695907440
77814794000
77688235280
7642264808

Consequently, the root required is 3·141599.

Ex. 4. The root is found to be between 1·1 and 1·2. Taking 1·1, its 5th power is 1·61051.
Hence $\text{G} = 2$, $n = 5$, $r = 1{\cdot}1$, $\text{A} = 1{\cdot}61051$,

$$\begin{array}{lr} \overline{n+1} \times \text{A} = 6 \times 1{\cdot}61051 = & 9{\cdot}66306 \\ \overline{n-1} \times \text{G} = 4 \times 2 = & 8 \\ \hline & 17{\cdot}66306 \end{array}$$

$$\begin{array}{lr} \overline{n+1} \times \text{G} = 6 \times 2 = & 12 \\ \overline{n-1} \times \text{A} = 4 \times 1{\cdot}61051 = & 6{\cdot}44204 \\ \hline & 18{\cdot}44204 \end{array}$$

As 17·66306 : 18·44204 : : 1·1
1·1

17·66306)20·286244(1·14
2623184
8568780
15035560
905112

Second Operation.

Here $G = 2$, $n = 5$, $r = 1·14$, $A = 1·9254145824$,

$$\overline{n+1} \times A = 6 \times 1·9254145824 = 11·5524874944$$
$$\overline{n-1} \times G = 4 \times 2 = 8$$
$$19·5524874944$$

$$\overline{n+1} \times G = 6 \times 2 = 12$$
$$\overline{n-1} \times A = 4 \times 1·9254145824 = 7·7016583296$$
$$19·7016583296$$

As 19·5524874944 : 19·7016583296 : : 1·14
1·14

7880663331184
2167182416256

19·5524874944)22·4598904957744(1·148696
290740300134
952154251904
1700347521280
1361485217280
1883359676160
1236358016640
63198766976

Therefore the root required is 1·148696.

Ex. 5. The root is found to be between 5·2 and 5·3.

Taking 5·2, its sixth power is 19770·609664.

Hence $G = 21035{\cdot}8$, $n = 6$, $r = 5{\cdot}2$, $A = 19770{\cdot}609664$,

$$\begin{array}{lr}
\overline{n+1} \times A = 7 \times 19770{\cdot}609664 = & 138394{\cdot}267648 \\
\overline{n-1} \times G = 5 \times 21035{\cdot}8 \qquad = & 105179 \\
\hline
 & 243573{\cdot}267648 \\
\hline
\end{array}$$

$$\begin{array}{lr}
\overline{n+1} \times G = 7 \times 21035{\cdot}8 \qquad = & 147250{\cdot}6 \\
\overline{n-1} \times A = 5 \times 19770{\cdot}609664 = & 98853{\cdot}04832 \\
\hline
 & 246103{\cdot}64832 \\
\hline
\end{array}$$

As $243573{\cdot}267648 : 246103{\cdot}64832 :: 5{\cdot}2$

$$\begin{array}{r}
246103{\cdot}64832 \\
5{\cdot}2 \\
\hline
49220729664 \\
123051824160 \\
\hline
\end{array}$$

$$243573{\cdot}267648)1279738{\cdot}971264(5{\cdot}25$$

$$\begin{array}{r}
618726330240 \\
97931611200
\end{array}$$

Second Operation.

Here $G = 21035{\cdot}8$, $n = 6$, $r = 5{\cdot}25$, $A = 20938{\cdot}99438476 5625$

$$\begin{array}{lr}
\overline{n+1} \times A = 7 \times 20938{\cdot}994384765625 = & 146572{\cdot}960693359375 \\
\overline{n-1} \times G = 5 \times 21035{\cdot}8 & 105179 \\
\hline
 & 251751{\cdot}960693359375 \\
\hline
\end{array}$$

$$\begin{array}{lr}
\overline{n+1} \times G = 7 \times 21035{\cdot}8 & = 147250{\cdot}6 \\
\overline{n-1} \times A = 5 \times 20938{\cdot}994384765625 = & 104694{\cdot}97192382125 \\
\hline
 & 251945{\cdot}57192382125 \\
\hline
\end{array}$$

As 251751·960693359375 : 251945·57192382125 : : 5·25
5·25

125972785961910625
50389114384764250
125972785961910625

251751·960693359375)1322714252600061562 5(5·254037
639544491332646875
1360405699459281250
1016458959924843750
945111715140625000
1898558330605468750
136284605751953125

Therefore the root required is 5·254037.

Ex. 6. The root is found to be between 1·1 and 1·2. Taking 1·1, its sixth power is 1·771561.

Hence $G = 2$, $n = 6$, $r = 1·1$, $A = 1·771561$,

$$\overline{n+1} \times A = 7 \times 1·771561 = 12·400927$$
$$\overline{n-1} \times G = 5 \times 2 \qquad = 10$$

22·400927

$$\overline{n+1} \times G = 7 \times 2 \qquad = 14$$
$$\overline{n-1} \times A = 5 \times 1·771561 = 8·857805$$

22·857805

As 22·400927 : 22·857805 : 1·1
1·1

22·400927)25·1435855(1·12
27426585
56256580
5454726

Second Operation.

Here $G = 2$, $n = 6$, $r = 1{\cdot}12$, $A = 1{\cdot}973822685184$

$$\begin{array}{lr} \overline{n+1} \times A = 7 \times 1{\cdot}973822685184 = & 13{\cdot}816758796288 \\ \overline{n-1} \times G = 5 \times 2 \qquad\qquad\qquad = & 10 \\ \cline{2-2} & 23{\cdot}816758796288 \\ \cline{2-2} \end{array}$$

$$\begin{array}{lr} \overline{n+1} \times G = 7 \times 2 \qquad\qquad\qquad = & 14 \\ \overline{n-1} \times A = 5 \times 1{\cdot}973822685184 = & 9{\cdot}86911342592 \\ \cline{2-2} & 23{\cdot}86911342592 \\ \cline{2-2} \end{array}$$

As $23{\cdot}816758796288 : 23{\cdot}86911342592 :: 1{\cdot}12$

$$\begin{array}{r} 1{\cdot}12 \\ \hline 4773822685184 \\ 2386911342592 \\ 2386911342592 \\ \hline \end{array}$$

$$\begin{array}{r} 23{\cdot}816758796288)26{\cdot}73340703370304(1{\cdot}122462 \\ 29166482407424 \\ 5349723611360 \\ 5863718518784 \\ 11003667595264 \\ 14769640767488 \\ 4795854897152 \\ 325031378944 \end{array}$$

Therefore the root required is 1·122462.

Ex. 7. The root is found to be between 4·1 and 4·2. Taking 4·1, its seventh power is 19475·4273881.

Hence $G = 21035{\cdot}8$, $n = 7$, $r = 4{\cdot}1$, $A = 19475{\cdot}4273881$,

$$\overline{n+1} \times A = 8 \times 19475{\cdot}4273881 = 155803{\cdot}4191048$$
$$\overline{n-1} \times G = 6 \times 21035{\cdot}8 \qquad = 126214{\cdot}8$$
$$\overline{282018{\cdot}2191048}$$

$$\overline{n+1} \times G = 8 \times 21035{\cdot}8 \qquad = 168286{\cdot}4$$
$$\overline{n-1} \times A = 6 \times 19475{\cdot}4273881 = 116852{\cdot}5643286$$
$$\overline{285138{\cdot}9643286}$$

As 282018·2191048 : 285138·9643286 : : 4·1

$$\begin{array}{r} 4{\cdot}1 \\ \hline 2851389643286 \\ 11405558573144 \\ \hline \end{array}$$

$$\begin{array}{l} 282018{\cdot}2191048)1169069{\cdot}75374726(4{\cdot}14 \\ \phantom{282018{\cdot}2191048)1}409968775552 \\ \phantom{282018{\cdot}2191048)1}1279505565046 \\ \phantom{282018{\cdot}2191048)11}151432683854 \end{array}$$

Second Operation.

Here $G = 21035{\cdot}8$, $n = 7$, $r = 4{\cdot}14$, $A = 20845{\cdot}02344116$,

$$\overline{n+1} \times A = 8 \times 20845{\cdot}02344116 = 166760{\cdot}18752928$$
$$\overline{n-1} \times G = 6 \times 21035{\cdot}8 \qquad = 126214{\cdot}8$$
$$\overline{292974{\cdot}98752928}$$

$$\overline{n+1} \times G = 8 \times 21035{\cdot}8 \qquad = 168286{\cdot}4$$
$$\overline{n-1} \times A = 6 \times 20845{\cdot}02344116 = 125070{\cdot}14064696$$
$$\overline{293356{\cdot}54064696}$$

As 292974·98752928 : 293356·54064696 :: 4·14
4·14

117342616258784
293356540646 96
117342616258784

292974·98752928)1214496·0782784144(4·145391
42596128161294
13298629408 3664
15796299071 9520
1147549695 48800
268624732900160
4947244219808

Therefore the root required is 4·145391.

Ex. 8. The root is found to be between 1·1 and 1·2. Taking 1·1, its 7th power is 1·9487171. Hence $G = 2$, $n = 7$, $r = 1{\cdot}1$, $A = 1{\cdot}9487171$,

$\overline{n+1} \times A = 8 \times 1{\cdot}9487171 = 15{\cdot}5897368$
$\overline{n-1} \times G = 6 \times 2 = 12$

27·5897368

$\overline{n+1} \times G = 8 \times 2 = 16$
$\overline{n-1} \times A = 6 \times 1{\cdot}9487171 = 11{\cdot}6923026$

27·6923026

As 27·5897368 : 27·6923026 :: 1·1
1·1

27·5897368)30·46153286(1·104089
2871796 76
1128231500
2464205600
2570267120
87190871

Therefore the root required is 1·104089.

Ex. 9. The root is found to be between 3·4 and 3·5. Taking 3·4, its eighth power is 17857·93904896.

Hence $G = 21035{\cdot}8$, $n = 8$, $r = 3{\cdot}4$, $A = 17857{\cdot}93904896$,

$\overline{n+1} \times A = 9 \times 17857{\cdot}93904896 = 160721{\cdot}45144064$

$\overline{n-1} \times G = 7 \times 21035{\cdot}8 = 147250{\cdot}6$

$307972{\cdot}05144064$

$\overline{n+1} \times G = 9 \times 21035{\cdot}8 = 189322{\cdot}2$

$\overline{n-1} \times A = 7 \times 17857{\cdot}93904896 = 125005{\cdot}57334272$

$314327{\cdot}77334272$

```
As 307972·05144064 : 314327·77334272 :: 3·4
                                        3·4
                    125731109337088
                    94298332002816
307972·05144064)1068714·429365248(3·47
                1447982750433 28
                 216094544670720
                    514108662272
```

Second Operation.

Here $G = 21035{\cdot}8$, $n = 8$, $r = 3{\cdot}47$, $A = 21020{\cdot}149394$,

$\overline{n+1} \times A = 9 \times 21020{\cdot}149394 = 189181{\cdot}344546$

$\overline{n-1} \times G = 7 \times 21035{\cdot}8 = 147250{\cdot}6$

$336431{\cdot}944546$

$\overline{n+1} \times G = 9 \times 21035{\cdot}8 = 189322{\cdot}2$

$\overline{n-1} \times A = 7 \times 21020{\cdot}149394 = 147141{\cdot}045758$

$336463{\cdot}245758$

```
As 336431·944546 : 336463·245758 : : 3·47
                                     3·47
                         -------------
                           23552427720306
                         13458529830320
                       1009389737274
                       --------------
336431·944546)1167527·46278026(3·470322
                1582316291422
                 2365885132386
                   1086152056400
                     768562227620
                      956983385280
                      284119496188
```

Therefore the root required is 3·470322.

Ex. 10. The root is found to be between 1 and 1·1. Taking 1, its eighth power is 1.

Hence $G = 2$, $n = 8$, $r = 1$, $A = 1$,

$$\overline{n+1} \times A = 9 \times 1 = 9$$
$$\overline{n-1} \times G = 7 \times 2 = 14$$
$$23$$

$$\overline{n+1} \times G = 9 \times 2 = 18$$
$$\overline{n-1} \times A = 7 \times 1 = 7$$
$$25$$

```
As 23 : 25 : : 1
            1
           --
     23)25·(1·08
          200
           16
```

Second Operation.

Here $G = 2$, $n = 8$, $r = 1\cdot08$, $A = 1\cdot85093131882$,

$\overline{n+1} \times A = 9 \times 1\cdot85093131882 = 16\cdot65838186938$

$\overline{n-1} \times G = 7 \times 2 = 14$

$30\cdot65838186938$

$\overline{n+1} \times G = 9 \times 2 = 18$

$\overline{n-1} \times A = 7 \times 1\cdot85093131882 = 12\cdot95651923174$

$30\cdot95651923174$

As 30·65838186938 : 30·95651923174 : : 1·08

1·08

24765215385408
30956519231760

30·65838186938)33·4330407703008(1·090502
27706589009208
15404532676600
7534174191000
1402497817124

Therefore the root required is 1·090502.

Ex. 11. The root is found to be between 3 and 3·1. Taking 3, its ninth power is 19683.
Hence $G = 21035\cdot8$, $n = 9$, $r = 3$, $A = 19683$,

$\overline{n+1} \times A = 10 \times 19683 = 196830$

$\overline{n-1} \times G = 8 \times 21035\cdot8 = 168286\cdot4$

$365116\cdot4$

$\overline{n+1} \times G = 10 \times 21035\cdot8 = 210358$

$\overline{n-1} \times A = 8 \times 19683 = 157464$

367822

As 365116·4 : 367822 :: 3
3

365116·4)1103466(3·02223
8116800
8144720
8423920
11215920
262428

Therefore the root required is found to be 3·02223.

Ex. 12. The root is found to be between 1 and 1·1.
Taking 1, its ninth power is 1.
Hence $G = 2$, $n = 9$, $r = 1$, $A = 1$,

$$\overline{n+1} \times A = 10 \times 1 = 10$$
$$\overline{n-1} \times G = 8 \times 2 = 16$$

26

$$\overline{n+1} \times G = 10 \times 2 = 20$$
$$\overline{n-1} \times A = 8 \times 1 = 8$$

28

As 26 : 28 :: 1
1

26)28(1·07
200
18

Second Operation.

Here $G = 2$, $n = 9$, $r = 1{\cdot}07$, $A = 1{\cdot}83845921242$,

$\overline{n+1} \times A = 10 \times 1{\cdot}83845921242 = 18{\cdot}3845921242$
$\overline{n-1} \times G = 8 \times 2 \qquad\qquad\qquad\quad = 16$

$$34{\cdot}3845921242$$

$\overline{n+1} \times G = 10 \times 2 \qquad\qquad\qquad\quad = 20$
$\overline{n-1} \times A = 8 \times 1{\cdot}83845921242 = 14{\cdot}70767369936$

$$34{\cdot}70767369936$$

As $34{\cdot}3845921242 : 34{\cdot}70767369936 :: 1{\cdot}07$
$1{\cdot}07$

$$\begin{array}{r} 24295371589552 \\ 34707673699360 \\ \hline \end{array}$$

$$\begin{array}{r} 34{\cdot}3845921242)37{\cdot}1372108583152(1{\cdot}0800\dot{5} \\ 2752618734115 \\ 1851364179200 \\ 132135572990 \end{array}$$

Consequently, the root required is found to be 1·08005.

To extract the Root of a Vulgar Fraction or mixed Number. (Page 45).

Ex. 1. The cube root of 8 = 2
do. 27 = 3
Therefore $\frac{2}{3}$ is the root required.

Ex. 2. 80)405(5
5)80(16
Hence $5|\frac{80}{405}| = \frac{16}{81}$ the fraction in its lowest terms.
The 4th root of 16 = 2
do. 81 = 3
Therefore $\frac{2}{3}$ = root required.

Ex. 3. $\frac{1}{2}$ = ·500. The cube root of this is soon found to be between ·79 and ·8.

Taking therefore ·79, its cube is ·493039.

Then, by the rule at page 41 of the Treatise,

·493039	·5
2	2
·986078	1
·5	·493039
1·486078	1·493039

As 1·486078 : 1·493039 : : ·79
·79
13437351
10451273
1·486078)117950081(·7937005 nearly
13924021
5499190
[illegible]
7014000

Ex. 4. $2\frac{10}{27} = \frac{64}{27}$.
The cube root of 64 = 4
do. 27 = 3
Therefore $\frac{4}{3}$ or $1\frac{1}{3}$ = root required.

Ex. 5. $7\frac{1}{5}$ = 7·2.
The cube root of this is soon found to be between 1·93 and 1·94.

Taking therefore 1·93, its cube is 7·189057.

Then, by the rule at page 41 of the Treatise,

7·189057	7·2
2	2
14·378114	14·4
7·2	7·189057
21·578114	21·589057

```
As 21·578114 : 21·589057 :: 1·93
                         1·93
                   ---------
                    64767171
                  194301513
                  21589057
                  ---------
21·578114)41·66688001(1·930978
          200887660
            66846341
             211199900
               169968740
                189219420
                 16594508
```

Therefore 1·930978 = root required.

DUODECIMALS. (Page 49).

```
Ex. 4.     4   7
           6   4
         ---------
          27   6
           1   6   4
          -----------
          29   0   4 = 29 f. 4 in.
          -----------

Ex. 5.    14   9
           4   6
         ---------
          59   0
           7   4   6
          -----------
          66   4   6 = 66 f. 54 in. *
          -----------
```

---

[*] That the correct answer to the above question is 66 sq. feet and 54 sq. inches, may be seen from the following considerations: the first product, namely, feet by feet, produces square feet; the next product, inches by

Ex. 6.	4	7	8	
	9	6		
	41	9	0	
	2	3	10	
	44	0	10	= 44 f. 10 in.

Ex. 7.	10	4	5			
	7	8	6			
	72	6	11			
	6	10	11	4		
		5	2	2	6	
	79	11	0	6	6	= 79 f. 132 in. 78″.

---

feet, produces rectangles of a foot in length, and an inch in width; the third product, inches by inches, produces square inches; and if we have a fourth product (as in Ex. 7), namely, inches by seconds, the result will be rectangles of an inch in length, and a second in width; a fifth product would produce square seconds, and so on. Therefore the number of sq. inches in the whole product is equal to 12 times the number of rectangles of a foot in length, plus the number of sq. inches: the number of sq. seconds is equal to 12 times the number of rectangles of an inch in length, plus the number of sq. seconds, &c. &c.

The truth of this statement may likewise be further seen by reducing each of the factors in the above example into inches, and finding their product. Thus,

14 f. 9 in. = 177 in. 4 f. 6 in. = 54 in.

The product of these is equal to 9558 sq. in., which being divided by 144 (inches in a sq. foot) gives 66 sq. feet 54 sq. in. as before.

Ex. 8.

$$\begin{array}{rrrrr}
39 & 10 & 7 & & \\
18 & 8 & 4 & & \\
\hline
717 & 10 & 6 & & \\
26 & 7 & 0 & 8 & \\
1 & 1 & 3 & 6 & 4 \\
\hline
745 & 6 & 10 & 2 & 4 \\
\hline
\end{array} = 745 \text{ f. } 82 \text{ in. } 28''.$$

Ex. 9.

$$\begin{array}{rrrrrr}
44 & 2 & 9 & 2 & 4 & \\
2 & 10 & 3 & & & \\
\hline
88 & 5 & 6 & 4 & 8 & \\
36 & 10 & 3 & 7 & 11 & 4 \\
 & 11 & 0 & 8 & 3 & 7 \\
\hline
136 & 2 & 10 & 8 & 10 & 11 \\
\hline
\end{array} = 136 \text{ f. } 34 \text{ in. } 106''.\ 11'''.$$

Ex. 10.

$$\begin{array}{rrrrrrr}
24 & 10 & 8 & 7 & 5 & & \\
9 & 4 & 6 & & & & \\
\hline
224 & 0 & 5 & 6 & 9 & & \\
8 & 3 & 6 & 10 & 5 & 8 & \\
1 & 0 & 5 & 4 & 3 & 8 & 6 \\
\hline
233 & 4 & 5 & 9 & 6 & 4 & 6 \\
\hline
\end{array} = 233 \text{ f. } 53 \text{ in. } 114''.\ 54'''.$$

MENSURATION OF SUPERFICIES.

PROBLEM I. (Page 95).

To find the Area of a Parallelogram.

Ex. 5.

35·25
35·25

17625
7050
17625
10575

10)1242·5625

124·25625
4

1·025
40

1·000

124 ac. 1 ro. 1 p. the area required.

Ex. 6.

12·25
8·5

6125
9800

10)104·125

10·4125
4

1·6500
40

26·0000

10 ac. 1 ro. 26 p. the area required.

Ex. 7. 12·5
9 in. = ·75 of a foot

625
875

$9·375 = 9\frac{375}{1000} = 9\frac{3}{8}$ feet.

Ex. 8. 37
$5\frac{1}{4}$

185
$9\frac{1}{4} = \frac{1}{4}$ of 37

$9)194\frac{1}{4}$ feet

$21\frac{7}{12}$ square yards.

PROBLEM II. (Page 97).

To find the Area of a Triangle.

Ex. 3. 40
30

2)1200

9)600 square feet

$66\frac{2}{3}$ yards.

Ex. 4.

First, 20 + 30 + 40 = 90, which divided by 2 = 45.

```
Then 45        45        45
     20        30        40
     --        --        --
     25        15         5         45
     --        --        --          5
                                   ---
                                   225
                                    15
                                  ----
                                  3375
                                    25
                                 -----
                                 16875
                                 6750
                                 -----
                                 84375
                                 -----

     84375(290·473
     4
    ---
   49)443
      441                10)290·473
      ---                   --------
   5804)27500               29·0473
        23216                     4
        -----               -------
   58087)428400               ·1892
         406609                  40
         ------              ------
   580943)2179100            7·5680
          1742829            ------
          -------
           436271
          -------
```

The area = 29 ac. 0 ro. 7 per.

Ex. 5.

$$\begin{array}{r} 25\tfrac{1}{4} = 25{\cdot}25 \\ 49 \\ \hline 22725 \\ 10100 \\ \hline 2)1237{\cdot}25 \\ \hline 9)618{\cdot}625 \\ \hline 68{\cdot}7361 \text{ sq. yards.} \\ \hline \end{array}$$

Ex. 6.

First, 380 + 420 + 765 = 1565, one half of which is 782·5.

Then

$$\begin{array}{rrr} 782{\cdot}5 & 782{\cdot}5 & 782{\cdot}5 \\ 380\phantom{{\cdot}5} & 420\phantom{{\cdot}5} & 765\phantom{{\cdot}5} \\ \hline 402{\cdot}5 & 362{\cdot}5 & 17{\cdot}5 \\ \hline \end{array}$$

$$\begin{array}{r} 782{\cdot}5 \\ 17{\cdot}5 \\ \hline 39125 \\ 54775 \\ 7825 \\ \hline 13693{\cdot}75 \\ 362{\cdot}5 \\ \hline 6846875 \\ 2738750 \\ 8216250 \\ 4108125 \\ \hline 4963984{\cdot}375 \\ \hline \end{array}$$

```
                4963984·375
                      402·5
              -------------
               24819921875
               9927968750
            198559375000
            ---------------
            1998003710·9375(44699·0347
   84         398
   886         6200 -
   8929         88437
   89389         807610
   8939803         31099375
   89398064         427996600
                     70404344

4840)44699·0347      (9·235337479 acres
        11390                    4
         17103       --------------
          25834      ·94135099172 roods
           16347                40
            18270     --------------
             37500   37·65403966880 poles
              36200  --------------
               23200
                38400
                 4520
                 ----
```

Ans. 9 ac. 0 ro. 37·65 poles.

Ex. 7.

feet in.
18 4
11 10

201 8
15 3 4

2)216 11 4

108 . 5 8 the area required.

Ex. 8.

First, 49·00 + 50·25 + 25·69 = 124·94, one half of which is 62·47,

62·47	62·47	62·47
49·00	50·25	25·69
13·47	12·22	36·78

62·47
36·78

40976
43729
37482
18741

2297·6466
12·22

45952932
45952932
45952932
22976466

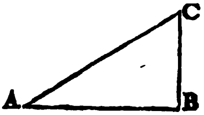
28077·241452

```
               28077·241452
                      13·47
               ------------
              196540690164
             112308965808
            84231724356
           28077241452
           ---------------
           378200·44235844(614·980
           36
           --
     121    182
            121
            ---
     1224    6100
             4896
             ----
     12289   120444
             110601
             ------
     122988    984323
               983904
               -------
                4195844
                -------
  10)614·980
     -------
      61·498
           4
     -------
       1·992
          40
     -------
       39·68
     -------
```

The area = 61 ac. 1 ro. 39·68 per.

PROBLEM III. (Page 98).

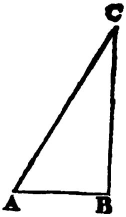
To find one Side of a right-angled Triangle, having the other two Sides given.



Ex. 3. By the annexed figure it is seen that the scaling ladder forms the hypothenuse, the height of the castle the perpendicular, and the breadth of the ditch the base of the right-angled triangle ABC. We have therefore the two legs given to find the hypothenuse: wherefore, by the rule,

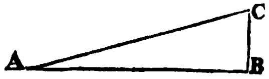
```
  28       45
  28       45
 ----     ----
 784      225
 ----    180
         ----
         2025
          784
         ----
         2809(53 = length of hyp. or ladder
         25
         ----
   103)  309
         309
         ----
```

Ex. 4. Here, by the figure, the shore forms the hypothenuse AC, the height of the jamb from the ground the perpendicular BC, and the distance of the foot of the shore from the bottom of the building, the base AB. Therefore, by the question, we have the two legs to find the hypothenuse.



```
 12        20
 12        20
----      ----
144        400
----       144
          ----
          544(23·32380 the hypothenuse,
          4       or length of the shore
          ----
     43   144
          129
          ----
    463    1500
           1389
           -----
   4662     11100
             9324
            ------
  46643      177600
             139929
             ------
 466468      3707100
             3731744
             -------
4664760        35356
             -------
```

Ex. 5. By the annexed figure, it appears that the line forms the hypothenuse AC; the height of the precipice the perpendicular BC;

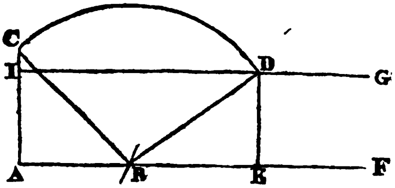


and the breadth of the brook, the base AB of the right-angled triangle ABC. We have, therefore, one leg and the hypothenuse to find the other leg. Then, by the rule,

```
   320          103
   320          103
  -----        -----
  6400          309
  960          1030
  ------       -----
  102400       10609
  ------       -----
        From  102400
        Take   10609
              ------
               91791(302·9703 feet = the breadth
               9                    of the brook
              ------
          602   1791
                1204
               -----
          6049   58700
                 54441
                -----
          60587   425900
                  424109
                  ------
          6059403   18910000
                    18178209
                    --------
                      731791
                    --------
```

Construction.

Ex. 6. Draw an indefinite line AF. At A make AC perpendicular to AF, and equal to the height of the moulding. With centre C and radius equal to the length of the ladder, cross AF at B.



On AC set off AI equal to the height of the window. Draw IG parallel to AF. With centre B and radius as before describe the arc crossing IG in D. Lastly, let fall the perpendicular DE: so shall AE be the breadth of the street.

Calculation.

In the triangle ABC we have the hypothenuse and one leg, to find the other. Wherefore, by the rule,

```
  28     50    From 2500
  28     50    Take  784
 ----   ----        ----
 784    2500        1716(41·4246303 = AB
                    16
                    ----
              81    116
                     81
                    ----
             824     3500
                     [illegible]
                     ----
            8282      20400
                      16564
                      -----
           82844       383600
                       331376
                       ------
          828486        5222400
                        4970916
                        -------
         8284923         25148400
                         24854769
                         --------
       828492603          2936310000
                          2485477809
                          ----------
                           450832191
                          ----------
```

Secondly, in the triangle BDE, we have given as in the last. Therefore,

```
  36          50
  36          50
 ----        ----
 1296   From 2500
 ----   Take 1296
             ----
             1204(34·698703 = BE
             9
             ----
      64     304
             256
             ----
      686     4800
              4116
              ----
      6929     68400
               62361
               -----
      69387     603900
                555096
                ------
      693967     4880400
                 4857769
                 -------
      69397403    226310000
                  208192209
                  ---------
                    8117791
                  ---------

41·4246303
34·698703
----------
76·1233333 = whole base or breadth of street.
----------
```

PROBLEM IV. (Page 99).

To find the Area of a Trapezoid.

Ex. 2.

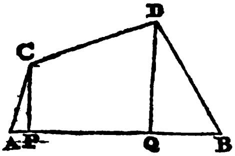
$$\begin{array}{rrrl}
\text{ft.} & \text{in.} & & \\
1 & 3 & & \rbrace = \text{breadths} \\
 & 11 & & \\
\hline
2 & 2 & & \\
12 & 6 & & = \text{length} \\
\hline
26 & 0 & & \\
1 & 1 & 0 & \\
\hline
2)27 & 1 & 0 & \\
\hline
13 & 6 & 6 & = 13\tfrac{78}{144} = 13\tfrac{13}{24} \text{ feet.} \\
\hline
\end{array}$$

(Page 100).

Ex. 3.

$$\begin{array}{rrrr}
21 & 3 & & \\
18 & 6 & & \\
\hline
39 & 9 & & \\
8 & 5 & & \\
\hline
318 & 0 & & \\
16 & 6 & 9 & \\
\hline
2)334 & 6 & 9 & \\
\hline
167 & 3 & 4 & 6 \\
\hline
\end{array}$$

Ans. 167 feet, 40 in. 6″.



Ex. 4. First for the area of the triangle APC, in which we have given the base AP, and perpendicular PC.

3·52
1·1
———
3·872 = double area of APC.

Secondly, for the area of the triangle QBD, in which we have given the perp. QD and the base QB.

AB − AQ = QB = 3·65
5·95
———
1825
3285
1825
———
21·7175 = double area of QBD.

Thirdly, for the area of the trapezoid CPQD, in which we have given the two parallel sides PC and QD, and the perpendicular distance between them PQ.

3·52
5·95

9·47
AQ — AP = PQ = 6·35

4735
2841
5682

60·1345 = double area of CPQD
21·7175
3·872

10)85·724 = double area of the whole in chains

2) 8·5724 = ditto in acres

4·2862 = 4 ac. 1 r. 5·792 perches, the area required.

PROBLEM V. (Page 102).

To find the Area of a Trapezium.

Ex. 4. 28
33·5

61·5
65

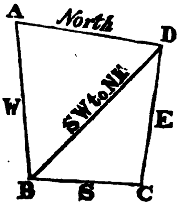
3075
3690

2)3997·5

9)1998·75

222·0833 or $222\frac{1}{12}$ sq. yards.

Ex. 5. By the annexed figure, it is plainly seen that the trapezium ABCD is composed of the two triangles ABDA and BDCB: In each of which we have given the three sides. First, for triangle BDCB, by rule at page 95 of the treatise.



48·35
35·75
27·40

2)111·50

55·75	55·75	55·75 = ½ sum of sides.
48·35	35·75	27·4
7·4	20	28·35 = the 3 remainders.

```
                 28·35
                 55·75
                ------
                 14175
                19845
               14175
              14175
             ---------
             1580·5125
                  7·40
            -----------
              632205000
            110635875
           ------------
           11695·792500
                     20
           ------------
          233915·850000(483·6484
          16
          ---
88          739
            704
           ----
963          3515
             2889
            -----
9666          62685
              57996
             ------
96724          468900
               386896
              -------
967288          8200400
                7738304
               --------
9672964          46209600
                 38691856
                ---------
                  7517744
                ---------
```

Secondly, for triangle ABDA,

```
  48·35
  37·55
  41·05
  -----
2)126·95
  ------
  63·475        63·475                  63·475
  48·35         37·55                   41·05
  ------        ------                  ------
  15·125        25·925                  22·425
  ------        ------                  63·475
                                        ------
                                        112125
                                       156975
                                      89700
                                     67275
                                   134550
                                  -----------
                                  1423·426875
                                       25·925
                                  -----------
                                   7117134375
                                  2846853750
                                12810844875
                                7117134375
                              2846853750
                              ---------------
                              36902·341734375
                              ---------------
```

```
                 36902·341734375
                          15·125
                ————————————————
                 184511708671875
                73804683468750
               36902341734375
             184511708671875
             36902341734375
             ———————————————————
             558147·918732421875(747·092978
             49
             ——
      144     681
              576
              ———
      1487     10547
               10409
               —————
      149409     1389187
                 1344681
                 ———————
      1494182      4450632
                   2988364
                   ———————
      14941849     146226842
                   134476641
                   —————————
      149418587      1175020118
                     1045930109
                     ——————————
      1494185948      12909000975
                      11953487584
                      ———————————
                        975513391
                      ———————————
```

$$\left.\begin{array}{l}483{\cdot}6484 = \text{area of BDCB}\\ 747{\cdot}092978 = \text{area of ABDA}\end{array}\right\}\text{in chains}$$

10)1230·741378

123·0741378 = area of the whole in acres
4

·2965512
40

11·8620480

The whole area 123 ac. 0 r. 11 per.

Ex. 6. 56·25
60·75

117·00
108·5

58500
93600
117000

2)12694·500

6347·25 or $6347\frac{25}{100}$, or $6347\frac{1}{4}$ feet.

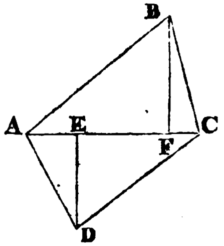
Ex. 7. 12
13
14
15

2)54

27 = ½ sum	27	27	27	27
	12	13	14	15
The 4 remainders	15	14	13	12

```
              13
              12
             ———
             156
              14
            ————
            2184
              15
           —————
           32760(180·997237 = the area required
           1
           ——
28         227
           224
           ———
3609        36000
            32481
            —————
36189        351900
             325701
             ——————
361987        2619900
              2533909
              ———————
3619942        8599100
               7239884
               ———————
36199443       135921600
               108598329
               —————————
361994467       2732327100
                2533961269
                ——————————
                 198365831
                ——————————
```

Ex. 8. Let the annexed figure ABCD represent the field. We have (preparatory to proceeding according to the rule), to find the perpendiculars BF and DE of the triangles BFCB and DEAD, by problem 3, at page 98 of the treatise.



First, for perpendicular BF,

265
265

1325
1590
530

70225 = sq. of hypothenuse BC

70
70

4900 = sq. of base FC

From 70225
Take 4900

65325(255·587 = perpendicular BF
4

45 253
225

505 2825
2525

5105 30000
25525

51108 447500
408864

511167 3663600
3578169

85431

Secondly, for perpendicular DE,

220 100
220 100
4400 10000 = sq. of base AE
440
48400 = sq. of hypothenuse DA.

From 48400
Take 10000
38400(195·954 = perp. DE
1
29 284
261
385 2300
1925
3929 37500
35361
39185 213900
195925
17975

Then, by the rule,

255·587
195·954
451·541 = sum of perpendiculars
378 = diagonal
3612328
3160787
1354623
2)170682·498
85341·849 = whole area in yards.

4840)8534·1849(17·632
36941
30618
15784
12649
2969

17·632 = area in acres
4

2·528
40

21·120

Answer 17 ac. 2 ro. 21 per.

PROBLEM VII. (Page 106).

To find the Area of a regular Polygon.

Ex. 2. 2·5980762 = tabular multiplier
400 = square of side

1039·23048 = answer.

Ex. 3. ·4330127 = tab. mult.
400 = sq. of side

173·20508 = answer.

Ex. 4. 4·8284271 = tab. mult.
400 = sq. of side

1931·37084 = answer.

Ex. 5. 7·6942088 = tab. mult.
400 = sq. of side

3077·68352 = answer.

PROBLEM VIII. (Page 107).

To find the Diameter and Circumference of a Circle, the one from the other.

Ex. 3. By rule 3,

As 1 : 3·1416 : : 7958 : 2500·8528
7958

251328
157080
282744
219912

25000·8528 the answer.

Ex. 4. By rule 3,

As 3·1416 : 1 : : 25000 : 7957¾ nearly,
3·1416)25000·0(7957·72, or 7957¾ nearly.
300880
181360
242800
228880
89680
26848

PROBLEM IX. (Page 108).

To find the Length of any Arc of a Circle.

Ex. 3. 12$\frac{1}{6}$ degrees = 12·1666,
Then, by rule 1,
3·1416
10 = radius

As 180 : 12·16666 :: 31·416
31·416

7299999
1216666
4866666
1216666
3649999

180)382·227892259(2·1234$\frac{8}{9}$ = answer
222
422
627
878
158

$\frac{158}{180} = \frac{79}{90} = \frac{8}{9}$ nearly.

(Page 109)

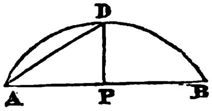
Ex. 4. By rule 2,
3·5 = chord of one-half the arc
8

28
6 = chord of the whole arc

3)22

7$\frac{1}{3}$ = length of the arc.

Ex. 5. Let the annexed figure ADB represent the arc, having its height PD 3, and its chord AB 8; that is, AP or PB 4.



First, the chord of one-half the arc AD forms the hypothenuse of the right-angled triangle ADP, and will be found by problem 3; viz. by taking the square root of the sum of the squares of AP and PD, which form the two legs of the triangle. Thus,

4
4
—
16 square of AP
—

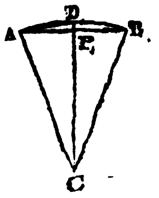
3
3
—
9 square of PD.
—

To 16
Add 9
—
25 the square root of which is 5, the chord of half the arc AD.

Then, by rule 2,

5
8
—
40
8 chord of whole arc
—
3)32
—
10$\frac{2}{3}$ length of arc.
—

Ex. 6. Let the annexed figure ADB represent the arc, having its chord AB 6; that is, AP or PB 3; and the radius of the generating circle AC, DC, or BC 9.



First, to find the base PC of the right-angled triangle APC, in which we have the perp. AP = 3, and hypothenuse AC 9 (by problem 3). Thus,

```
 9                 3
 9                 3
--                --
81 sq. of AC.      9 sq. of AP.
--                --

      From 81
      Take  9
           --
           72(8·48528 = PC
           64
           --
      164   800
            656
           ----
      1688  14400
            13504
           ------
      16965   89600
              84825
             ------
      169702   477500
               339404
              -------
      1697048  13809600
               13576384
               --------
                 233216
               --------
```

The height PD may now be found by taking PC from the radius DC: thus,

```
From 9          = DC
Take 8·48528    = PC
     -------
      ·51472    = PD.
     -------
```

Then, to find the chord of one-half the arc AD, we have, as in the last example, two sides of the right-angled triangle APD, namely, AP = 3, and PD = ·51472, to find the third side AD: thus,

·51472
·51472

102944
360304
205888
51472
257360

·2649366784 = sq. of PD
9 = sq. of AP

9·2649366784(3·043835 = AD the chord of ½ the arc
9

604 2649
2416

6083 23336
18249

60868 508767
486944

608763 2182384
1826289

6087865 35609500
30439325

5170175

Then, by the rule,
3·043835
8

24·350680
6 = chord of the arc

3)18·350680

6·11689 the answer.

PROBLEM X. (Page 110).

To find the Area of a Circle.

Ex. 2. By rule 1,

22
7
4)154
38½ answer

Ex. 3. 4)3·1416 = product of diam. and circum.
·7854 answer.

Ex. 4. By rule 2,

7
7
49 = sq. of diameter

Then ·7854
49
70686
31416
38·4846 answer.

Ex. 5. By rule 2,

3·5
3·5
175
105
12·25 sq. of diam.

Then ·7854
12·25
39270
15708
15708
7854
9)9·621150 area in feet
1·069016 answer.

Ex. 6. By rule 5,

10·9956
10·9956

659736
549780
989604
989604
1099560

120·90321936 = sq. of circumference.

Then, as 88 : 7 :: 120·90321936
7

88)846·32253552(9·61730154
543
152
642
265
135
475
352

9·61730154 area in yards
9

86·55571386 area in feet.

PROBLEM XI. (Page 112).

To find the Area of the Sector of a Circle.

Ex. 3. 10 = radius
10 = ½ the arc

100 the area

Ex. 4. By the sixth example of the ninth problem, the length of the arc is found to be 6·11689. Then, by the rule,

2)6·11689
3·058445 = one-half arc
9 = radius
27·526005 the answer.

Ex. 5. The area of the whole circle is found by rule 2, problem 10: thus,

25 radius
2
50 diameter
50
2500
·7854
2500
392700
15708
1963·500 area of the whole circle.

Also 147 degrees 29 min. = 147·48333 degrees.

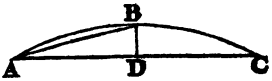
Then by rule 2,

```
          As 360 : 147·48333 : : 1963·5
                              1963·5
                           ---------
                            73741666
                           44245000
                          88490000
                         132735000
                         14748333
                         ------------
      360)289583521666(804·3986 the answer.
              1583
               1435
                3552
                 3121
                   2416
                    216

Ex. 6.              13
                     2
                    --
                    26 = diameter
                    26
                   ---
                   156
                   52
                   ---
                   676 = square of diameter
                   ---
                 ·7854
                   676
                 -----
                 47124
                54978
               47124
               -------
       2)530·9304 = area of the whole circle
       ----------
       2)265·4652    do.  -  -  -  semicircle
       ----------
         132·7326    do.  -  -  -  quadrant
         --------
```

PROBLEM XII. (Page 114).

To find the Area of a Segment of a Circle.

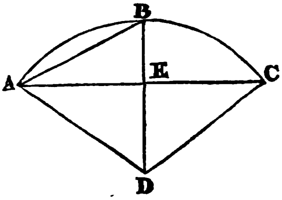


Ex. 2. Let ABCD be the segment, the area of which is required. First, AB^2 is equal to the sum of the squares of AD and BD: and $AB^2 \div BD =$ diameter of the whole circle. Thus,

```
 10      2        52)2·00(·038 and
  10     2        ————
————    ——    To ·038 answers in the table
 100     4        ·009763       Then
   4                            ·009763
2)104              sq. of diam. = 2704
————                            ————
  52 = diameter                 39052
                               683410
                              19526
                              ————————
                              26·309152 the ans.
                              ————————
```

Ex. 3. By rule 3,

```
50)18
 ————
   ·36 to which answers, in the table, ·254550.
          Then ·254550
sq. of diam. =    2500
           ———————————
           636·375000 the answer.
           ———————————
```



Ex. 4. Let the annexed figure ABCA be the segment, whose area is required: also, let D be the centre of the circle, and AD, or BD, or CD, the radius. Find the perpendicular in the right-angled triangle AEDA, in which are given the base AE 8, and hypothenuse AD = 10. Thus (problem 3, rule 2),

10 8
10 8

100 = sq. of AD 64 = sq. of AE.

From 100
Take 64

36 the square root of which is 6 = the line DE,

And 10 − 6 = 4 = BE, the height of the segment.

Then, by the rule,

20)4

·2 to which answers in the table ·111823

sq. of diam. = 400

44·7292 the answer.

PROBLEM XIII. (Page 115).

To find the Area of a Circular Ring, or Space included between two Concentric Circles.

Ex. 2. 20 20
10 10

30 = sum of diam. 10 = diff. of diam.

·7854
30

23·5620
10

235·62 the answer.

Ex. 3.

```
 6                      6
 4                      4
 —                      —
10 = sum of diam.       2 = diff. of diam.
 —                      —
          ·7854
             10
          ——————
          7·854
              2
          ——————
         15·708  the answer.
         ——————
```

PROBLEM XIV. (Page 116).

To measure long Irregular Figures.

Ex. 2.

```
  17·4
  24·4
  ————
2)41·8
  ————
  20·9 mean breadth of the first and last
  20·6
  14·2
  16·5
  20·1
  ————
5)92·3
  —————
  18·46 mean breadth of all
     84 length
  ———————
   7384
 14768
 ———————
 1550·64 the answer.
 ———————
```

MENSURATION OF SOLIDS.

PROBLEM I. (Page 121).

To find the Solidity of a Cube.

Ex. 2.

```
  22 feet
  22
  --
  44
 44
 ---
 484
  22
 ---
 968
968
-----
10648 the answer.
-----
```

Ex. 3. 18 inches = $\frac{3}{2}$ foot

$\frac{3}{2} \times \frac{3}{2} \times \frac{3}{2} = \frac{27}{8} = 3\frac{3}{8}$ the answer.

PROBLEM II. (Page 122).

To find the Solidity of a Parallelopipedon.

Ex. 2.

```
  10·5 = length
   4·2 = breadth
  ----
   210
  420
  -----
  44·10
    3·4 = height
  -----
  17640
 13230
 -------
 149·940 the answer.
 -------
```

Ex. 3.

$$\begin{array}{rrr}
\text{f.} & \text{in.} & \\
3 & 2 & \\
2 & 8 & \\
\hline
6 & 4 & \\
2 & 1 & 4 \\
\hline
8 & 5 & 4 \\
2 & 6 & \\
\hline
16 & 10 & 8 \\
4 & 2 & 8 \\
\hline
21 & 1 & 4 = 21\frac{1}{9}\text{ feet.} \\
\hline
\end{array}$$

PROBLEM III. (Page 123).

To find the Solidity of any Prism.

Ex. 2. The area of the triangular end will be found by problem 2, rule 2, of Mensuration of Superficies: thus,

$$\begin{array}{r}
5 \\
4 \\
3 \\
\hline
2)12 \\
\hline
6 = \frac{1}{2}\text{ sum of sides}
\end{array}
\qquad
\begin{array}{rrr}
\text{From } 6 & 6 & 6 \\
\text{Take } 5 & 4 & 3 \\
\hline
1 & 2 & 3 \\
 & & 2 \\
\hline
 & & 6 \\
 & & 6 \\
\hline
\end{array}$$

$$\sqrt{36} = 6 \text{ the area of the end}$$

$$\begin{array}{l}
\text{Then } 6 \\
\quad 10 = \text{length} \\
\hline
\quad 60 \text{ the answer.} \\
\hline
\end{array}$$

Ex. 3. The area of the hexagonal end is to be found by problem 7, rule 2, of Mensuration of Superficies: thus:

First 1 f. 6 in. = 1·5, and $1{\cdot}5^2 = 2{\cdot}25$,

Then 2·5980762 = tabular multiplier
2·25 = square of side

129903810
51961524
51961524

5·84567145 = area of the end
8 = length

46·76537160 = content

Ex. 4. The area of the circular end is to be found by problem 10, rule 3, of Mensuration of Superficies: thus:

5½ feet = 5·5 and $5{\cdot}5^2 = 30{\cdot}25$,

Then ·07958
30·25

39790
15916
238740

2·4072950 = area of end
20 = length

48·1459 = content.

Ex. 4. First, 2 f. 3 in. = $2\frac{1}{4}$ feet = $\frac{9}{4}$ feet.
$(\frac{9}{4})^2 = \frac{81}{16}$ $\frac{81}{16} \times 16$ (length or height) $= \frac{81}{16} \times \frac{16}{1} = 81$

$$\begin{array}{rl} \cdot 7854 & \\ 9 & \times\ 9 = 81 \\ \hline 7\cdot 0686 & \\ 9 & \\ \hline 63\cdot 6174 & = \text{content.} \\ \hline \end{array}$$

PROBLEM IV. (Page 124).

To find the Convex Surface of a Cylinder.

Ex. 2.

$$\begin{array}{rl} 3\cdot 1416 & \\ 2 & \\ \hline 6\cdot 2832 & = \text{circum. of end} \\ 20 & = \text{length} \\ \hline 125\cdot 664 & = \text{content.} \\ \hline \end{array}$$

Ex. 3.

$$\begin{array}{rrl} \text{f.} & \text{in.} & \\ 18 & 6 & \\ 5 & 4 & \\ \hline 96 & 6 & \\ 6 & 2 & \\ \hline 98 & 8 & = 98\frac{2}{3} \text{ feet.} \\ \hline \end{array}$$

PROBLEM V. (Page 125).

To find the Convex Surface of a Right Cone.

Ex. 2.

20

9

2)180

90 = the answer.

Ex. 3. First, 8 f. 6 in. = 8·5.

3·1416

8·5

157080

251328

26·70360 = circum.

50

2)1335·18

667·59 = content.

PROBLEM VI. (Page 125).

To find the Convex Surface of the Frustrum of a Right Cone.

Ex. 2.

6
8·4

14·4 = sum of perimeters
12·5

720
288
144

2)180·00

90 = content.

Ex. 3.

f. in.
2 3
5 4

7 7
10 6

75 10
3 9 6

2)79 7 6

39 9 9 = 39 sq. f. 117 in. = 39$\frac{13}{16}$ the ans.

PROBLEM VII. (Page 127).

To find the Solidity of the Frustrum of a Cone.

Ex. 3. The area of the base is found by problem 10, rule 3, of Mensuration of Superficies: thus,

$$\begin{array}{r l} \cdot07958 & \\ 81 & = \text{sq. of circum.} \\ \hline 7958 & \\ 63664 & \\ \hline 6\cdot44598 & = \text{area of base} \\ 10\frac{1}{2} & \\ \hline 64\cdot45980 & \\ 3\cdot22299 & = \text{one-half of } 6\cdot44598 \\ \hline 3)67\cdot68279 & \\ \hline 22\cdot56093 & = \text{content.} \\ \hline \end{array}$$

Ex. 4. The area of the base is found by problem 2, rule 2, of Mensuration of Superficies: thus,

$$\begin{array}{r l} 5 & \\ 6 & \\ 7 & \\ \hline 2)18 & \\ \hline 9 & = \text{one-half sum of sides} \end{array}$$

$$\begin{array}{r r r r} \text{From} & 9 & 9 & 9 \\ \text{Take} & 5 & 6 & 7 \\ \hline & 4 & 3 & 2 \\ & 3 & & \\ \hline & 12 & & \\ & 2 & & \\ \hline & 24 & & \\ & 9 & & \\ \hline & 216 & & \end{array}$$

```
Then,         216(14·696937 = area of the base
              1
              ---
24            116
               96
              ---
286            2000
               1716
               ----
2920            28400
                26361
                -----
29386            203900
                 176316
                 ------
293929            2758400
                  2645361
                  -------
2939383            11303900
                    8818149
                   --------
29393867            248575100
                    205757069
                    ---------

               14·696937
                    14·5 = height
               ---------
                73484685
               58787748
              14696937
              ---------
            3)213·1055865
              -----------
               71·0351955 the content.
              -----------
```

Ex. 5. The area of the base is found by problem 7, rule 2, of Mensuration of Superficies: thus,

2·5980762 = tabular number
·25 = square of 6 inches, or ·5

129903810
51961524

·649519050 = area of base
6·4 = height

2598076200
3897114300

3)4·1569219200

1·38564064 = the content.

PROBLEM VIII. (Page 129).

To find the Solidity of the Frustrum of a Cone, or any Pyramid.

	10	10
400	100	200

100
400
200

700

·026526 = tabular multiplier
700

18·568200 = mean area
25

92841000
37136400

464·205000 the answer.

Ex. 6. First, 1 f. 3 in. $= 1\frac{1}{4} = \frac{5}{4}$ foot.

6 in. $= \frac{1}{2}$ foot.

$$\left(\tfrac{5}{4}\right)^2 = \tfrac{25}{16}$$

$$\left(\tfrac{1}{2}\right)^2 = \tfrac{4}{16} \text{ or } \tfrac{1}{4}$$

$$\tfrac{5}{4} \times \tfrac{1}{2} = \tfrac{10}{16} \text{ or } \tfrac{5}{8}$$

The sum of these is equal to $\frac{39}{16}$, which being divided by 3 (or multiplied by $\frac{1}{3}$), will give the mean area to be multiplied by the length. Thus,

$$\tfrac{39}{16} \times \tfrac{1}{3} \times \tfrac{24}{1} = \tfrac{39}{16} \times 8 = \tfrac{39}{2} = 19\tfrac{1}{2} \text{ feet.}$$

```
Ex. 5.   8    4    8           ·2618
         8    4    4            112
        --   --   --         -------
        64   16   32           5236
        --   --   64          28798
                  16         -------
                 ---          29·3216 mean area
                 112               18 altitude
                            ---------
                             527·7888 the answer.
                            ---------
```

```
Ex. 6.    First, 18 inches = 1·5 feet.
                 12        = 1.

           Then 1·5       1       1·5
                1·5       1       1
                ----     --      ----
                2·25      1       1·5
                1
                1·5
                ----
                4·75
                ----
```

Then,

2·5980762 = tab. number
4·75

129903810
181865334
103923048

3)12·340861950

4·11362065
6

26·6817239 the answer.

PROBLEM IX. (Page 130).

To find the Solidity of a Wedge.

Ex. 2. 70
2

140
110

250 = sum to be reserved.

Then 34·29016
30

1028·7048
250

6)257176·20000

1728)42862·70000(24·8048 = content
8302
13907
8300
13880
56

PROBLEM X. (Page 132).

To find the Solidity of a Prismoid.

Ex. 2.	12	8	12
	8	6	8
	96	48	2)20

10 length of middle section

6
8

2)14

7 = breadth of middle section
10

70 area of middle section.
4

280
96
48

6)424

70·666 +
60

1728)423·9999(2·453 the answer.
7839
9279
6399
1215

```
Ex. 3.    81·5        29·5
            55          41
          ----        ----
          4075         295
         4075         1180
         ------       ------
         4482·5       1209·5
         ------       ------
          81·5          55
          41            29·5
          ----          ----
       2)122·5       2)84·5
          ----          ----
          61·25         42·25 breadth of mid. section
          ----          61·25 length of do.
                        -----
                        21125
                        8450
                       4225
                      25350
                      -------
                      2587·8125=area of middle section
                              4
                      ----------
                     10351·25
                      4482·5
                      1209·5
                     --------
                   6)16043·25
                     --------
                      2673·875 mean area
                        47·25 length
                      --------
                      13369375
                      5347750
                    18717125
                   10695500
                   ------------
                   126340·59375 the answer.
                   ------------
```

PROBLEM XI. (Page 132).

To find the Convex Surface of a Sphere.

Ex. 2.

22
7

154 the answer.

(Page 133).

Ex. 3.

7957·75
25000

3978875000
1591550

198943750 miles.

Ex. 4.

3·1416
42 diameter

62832
125664

131·9472 circumference
9 height

1187·5248 answer.

Ex. 5.

$$\begin{array}{r}
3{\cdot}1416 \\
2 \\
\hline
6{\cdot}2832 \\
12\tfrac{1}{2} \\
\hline
75{\cdot}3984 \\
3{\cdot}1416 \\
\hline
78{\cdot}54 \\
\hline
\end{array}\quad \text{the answer.}$$

PROBLEM XII. (Page 133).

To find the Solidity of a Sphere.

Ex. 2. First, 2 feet 8 in. = $2\frac{2}{3}$ feet = $\frac{8}{3}$ feet.

Then $\frac{8}{3} \times \frac{8}{3} \times \frac{8}{3} = \frac{512}{27}$, cube of diameter or axis.

And, by the rule,

$$\begin{array}{r}
{\cdot}5236 \\
512 \\
\hline
10472 \\
5236 \\
26180 \\
\hline
27)268{\cdot}0832 \\
\hline
9{\cdot}929 \\
\hline
\end{array}\quad \text{answer.}$$

Ex. 3. First, by rule 3, problem 8, of Mensuration of Superficies,

```
3·1416)25000·0(7957·72 = 7957¾ nearly
            300880
             181360
              242800
               228880
                89680
                26848
   7957¾ = 7957·75 the diameter or axis.
            Then 7957·75 *
                 7957·75
                 -------
                 3978875
                5570425
               5570425
              3978875
             7161975
            5570425
            --------------
            63325785·0625
                  7957·75
            --------------
            3166289253125
           4432804954375
          4432804954375
         3166289253125
        5699320655625
       4432804954375
       ---------------------
        503930766081·109375  cube of axis
                      ·5236
       ---------------------
       3023584596486656250
      1511792298243328125
     1007861532162218750
    2519653830405546875
    ---------------------
    263858149120·06886875  cubic miles.
    ---------------------
```

---

* The diameter must be considered as 7957¾ miles, or the result will not agree with that in the Treatise.

PROBLEM XIII. (Page 134).

To find the Solidity of a Spherical Segment.

Ex. 2.

$$\begin{array}{rl}
2)20 & = \text{diameter} \\
\hline
10 & = \text{radius} \\
10 & \\
\hline
100 & \\
3 & \\
\hline
300 & \\
\hline
 & \\
9 & \\
9 & \\
\hline
81 & = \text{square of height} \\
300 & \\
\hline
381 & \\
9 & = \text{height} \\
\hline
3429 & \\
\cdot 5236 & \\
\hline
20574 & \\
10287 & \\
6858 & \\
17145 & \\
\hline
1795\cdot 4244 & \text{answer.} \\
\hline
\end{array}$$

Ex. 3.

```
        2)8·61684
          -------
           4·30842 = radius of the base
           4·30842
           -------
            861684
          1723368          2·25
        12925260           2·25
       1723368             ----
       -------------       1125
       18·5624828964       450
                   3      450
       -------------     -----
       55·6874486892      5·0625  sq. of height.
        5·0625           ------
       -------------
       60·7499486892
       -------------

                        60·7499486892
                                 2·25
                       --------------
                        3037497434460
                       1214998973784
                      1214998973784
                      ---------------
                      136·6873845550700
                                  ·5236
                      -----------------
                      8201243073042
                     4100621536521
                    2733747691014
                   6834369227535
                   -----------------
The answer         71·56951455074652
                   -----------------
```

PROBLEM XIV. (Page 135).

To find the Solidity of a Spherical Zone or Frustrum.

Ex. 2. 2)12 2)10

6 and 5 = the radii of the two ends
6 5

36 25

2
2

3) 4

1⅓ = ⅓ of distance or height
25
36

62⅓

1·5708
62⅓

31416
94248
·5236 = ⅓ of the multiplicand 1·5708

97·9132
2 = height

195·8264 the answer.

Ex. 3.

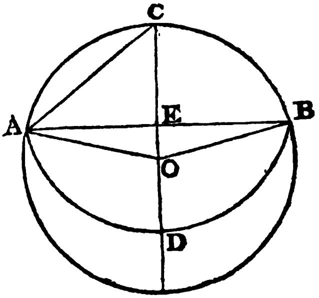
2)6
3 the radius of each end.
3
9

8
8
3)64
$21\frac{1}{3}$ = one-third of height.
9
9
$39\frac{1}{3}$

1·5708
$39\frac{1}{3}$
141372
47124
·5236 = one-third of 1·5708
61·7848
8 = height
494·2784 the answer.

PROBLEM XV. (Page 137).

To find the Surface of a Circular Spindle.



Ex. 2. Let the annexed figure ACBDA represent the circular spindle, having its length AB = 24, and thickness CD = 18. Now $AE^2 = 144$, and $CE^2 = 81$; and $144 + 81 = 225 = AC^2$ by prob. 3 of Mensuration of Superficies. Also, 225, or $AC^2 \div 9$ or CE, = 25 = the diameter of the generating circle CF; or the radius CO = 12·5. Hence, the central distance OE = OC − CE = 12·5 − 9 = 3·5. Also, by problem 10, rule 2, of Mensuration of Superficies,

$\sqrt{225} = 15$ = AC the chord of one-half the arc
8

120
24 = AB the chord of the whole arc

3)96

32 = arc ACB.

Then, by the rule, 12·5 = radius
24 = length

500
250

300·0 = former product.

```
Again,      32 = arc ACB
           3·5 = central distance
          ----
           160
           96
          ----
Take       112 latter product
From       300 former
          ----
Remains    188
          ----
                6·2832
                  ·188
               -------
                502656
               502656
               62832
              --------
             1181·2416 the answer.
              --------
```

(Page 138).

To find the Solidity of a Circular Spindle.

Ex. 2. Let the figure ABCDA in the last problem represent the circular spindle, having its length 24, and thickness 18. Also, by the work in the last example, the central distance OE was found to be 3·5, the diameter CF 25, and the arc ACB 32. Then the area of the revolving segment ACBA is found by problem 12, rule 1, of Mensuration of Superficies,

```
        32 = arc
        25 = diam.
       ----
       160
       64
       ----
     4)800
       ----
       200 = area of the sector ACBOA.
```

Again, 24 = AB the base } of the triangle ABOA
3·5 = OE the perp. }

120
72

2)84·0

Take 42 = area of triangle ABOA
From 200 = area of sector, as before found

158 = area of the revolving segment ACBA
1·75 = one-half central distance

790
1106
158

Take 276·50
From 576

299·5

AE = 12
12

144
12

3)1728

576

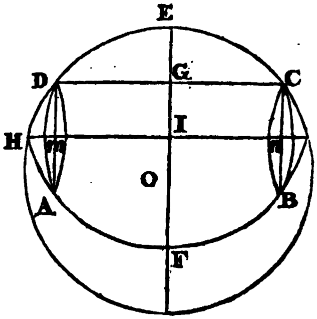
12·5664
299·5

628320
1130976
1130976
251328

The answer 3763·6368

PROBLEM XVII. (Page 140).

To find the Solidity of the middle Frustrum of a Circular Spindle.



Ex. 2. Let DECBFAD be the frustrum: the length *mn* being 20, the greater diameter EF = 18, and the less diameter DA or CB = 8. Draw DG parallel to *m*I, and we shall have DG = *m*I = 10. And D*m* being equal to GI, EG will be equal to EI less D*m* or GI: that is, EG = 9 − 4 = 5. And $DG^2 + EG^2 = 100 + 25 = 125 = DE^2$. Also, $DE^2 \div EG = 125 \div 5 = 25 =$ diam. or the radius OE = 12·5. Again, OE−IE, or 12·5 − 9 = 3·5 = OI the central distance. And one-half the length of the whole spindle, or HI, is found by problem 3 of Mensuration of Superficies. Thus, $HO^2 - OI^2 = 152{\cdot}25 - 12{\cdot}25 = 144 = HI^2$. Then, by the rule,

$$\begin{array}{rl}
 & 144 \\
\tfrac{1}{3} \text{ of } mI^2\text{, or } \tfrac{1}{3} \text{ of } 100 = & 33\tfrac{1}{3} \\
\hline
 & 110\tfrac{2}{3} \\
 & 10 \quad = \tfrac{1}{2} \text{ length of frustrum} \\
\hline
 & 1106\tfrac{2}{3} = \text{former product.} \\
\hline
\end{array}$$

Again, to find the revolving area generating the middle frustrum, which is composed of the segment DECGD, and of the rectangle DC*nm*D. First, we proceed to find the area of the former, by rule 3, problem 12, of Mensuration of Superficies,

EG ÷ 2OE (the diam.) = 5 ÷ 25 = ·2.

To which corresponds in the table	·111823
Square of diam.	625
	559115
	223646
	670938
area of seg.	69·889375
DM × *mn* = 4 × 20 = 80 area of rect.	80
	149·889375
central distance	3·5
	749446875
	449668125
Take latter product	524·6128125
From former	1106·6666666
	582·0538541
	6·2832
	11641077082
	17461615623
	46564308328
	11641077082
	34923231246
The answer	3657·16077608112

PROBLEM XVIII. (Page 142).

To find the Superficies or Solidity of any regular Body.

Ex. 2.
2
2
4
6 = tabular number
24 = superficial content

2
2
4
2
8 = solidity.

Ex. 3.
2
2
4
3·4641 = tabular number
4
13·8564 = superficies

2
2
4
2
8
·4714 = tabular number
8
3·7712 = solidity.

Ex. 4. 2

2

4

20·64573 = tabular number

4

82·58292 = superficies

2

2

4

2

8

7·66312 = tabular number

8

61·30496 = solidity.

Ex. 5. 2

2

4

8·66025 = tabular number

4

34·64100 = superficies

2

2

4 2·18169 = tab. number

2 8

8 17·45352 = solidity.

PROBLEM XIX. (Page 143).

To find the Surface of a Cylindrical Ring.

Ex. 2.
4
16
20
4
80
9·8696
80
789·5680 the answer.

PROBLEM XX. (Page 144).

To find the Solidity of a Cylindrical Ring.

4
16
20
16 = square of thickness
320
2·4674
320
493480
74022
789·5680 the content.

TIMBER MEASURING.

PROBLEM I. (Page 149).

To find the superficial Content of a Board or Plank.

Ex. 2.

$$\begin{array}{rrrl} \text{f.} & \text{in.} & & \\ 11 & 2 & & \\ 1 & 10 & & \\ \hline 11 & 2 & & \\ 9 & 3 & 8 & \\ \hline 20 & 5 & 8 & = 20 \text{ f. } 68 \text{ in. the answer.} \\ \hline \end{array}$$

Ex. 3.

$$\begin{array}{rrrl} \text{f.} & \text{in.} & & \\ 12 & 9 & & \\ 1 & 3 & & \\ \hline 12 & 9 & & \\ 3 & 2 & 3 & \\ \hline 15 & 11 & 3 & = 15 \text{ f. } 135 \text{ in.} \\ \hline \end{array}$$

$$\begin{array}{r|rl} 6 \text{ in. is } \frac{1}{2} & & 2\frac{1}{2}d. \\ 4 \text{ in. is } \frac{1}{3} & & 15 \\ \cline{2-3} & 3 & 1\frac{1}{2} \\ & & 1\frac{1}{4} \\ 1 \text{ in. is } \frac{1}{4} & & \frac{3}{4} \\ & & \frac{1}{4} \text{ nearly} \\ \cline{2-3} & 3 & 3\frac{3}{4} \text{ the answer.} \\ \cline{2-3} \end{array}$$

Ex. 4.

f.	in.	'	
1	1	6	breadth of first two
17	6		length
19	1	6	
	6	9	
19	8	3	
		2	
39	4	6	content of first two.

f.	in.	'	
1	2	6	breadth of third
17	6		length
20	6	6	
	7	3	
21	1	9	content of third.

in.							
18							
11	3						
2)29	3						
			f.	in.	"		
14	7	or	1	2	7		breadth of last two
			17	6			length
			20	7	11		
				7	3	6	
			21	3	2	6	
					2		
			42	6	5		content of last two.

f.	in.	"	
42	6	5	
21	1	9	
39	4	6	
103	0	8	content of the whole.

d.
3
103
12)309
20)25 9
1 5 9 value of the whole.

PROBLEM II. (Page 151).

To find the solid Content of squared or four-sided Timber.

Ex. 2.

24·5 length
1·04 breadth
980
2450
25·480
1·04 thickness
101920
254800
26·49920 = 26½ feet.

Ex. 3. $19\frac{1}{8}$
$9\frac{7}{8}$

2)29

14·5 mean length of two ends
14·5

725
580
145

144)210·25 mean area of two ends

1·46 do. in feet.

20·38 length
1·46

12228
8152
2038

29·7548 content required.

Ex. 4. 1·78   1·23
1·04   ·91

2)2·82   2)2·14

1·41 mean breadth   1·07 mean thickness.

1·41

1·07

987

1410

1·5087

27·36 length

90522

45261

105609

30174

41·278032 content in feet.

PROBLEM III. (Page 152).

To find the Solidity of round or unsquared Timber.

Ex. 2. 14

2

2)16

4)8 = mean girt of two ends

2 = $\frac{1}{4}$ girt

2

4

24 = length

96 feet *.

---

* It ought to be observed, that this rule, which generally gives the answer sufficiently correct for most practical

Ex. 3.

9·43
7·92
6·15
4·74
3·16

5)31·40

Mean girt 6·28

4)6·28

1·57 = ¼ girt
1·57

1099
785
157

2·4649
17¼

41·9033
·6162 = ¼ of 2·4649

42·5195 the answer.

Ex. 4.

4)3·15

·7875 = ¼ girt
·7875

39375
55125
63000
55125

·62015625
14·5 = length

310078125
248062500
62015625

8·992265625 content.

---

purposes, is not always to be relied on; and that it is only when the sides of a beam differ but an inch or two from each other that it may be used without inducing any material error.

RULE II. (Page 154).

Ex. 2. 5)8
1·6 = $\frac{1}{5}$ girt
1·6
2·56
48 = double the length
2048
1024
122·88 feet.

Ex. 3. 5)3·15
·63 = $\frac{1}{5}$ girt
·63
·189
378
·3969
29 = double the length
35721
7938
11·5101 the answer.

Ex. 4. 5)6·28
1·256 = $\frac{1}{5}$ girt
1·256
7536
6280
15072
1·577536
34·5 = double the length
7887680
6310144
4732608
54·4249920 the answer.

BRICKLAYERS' WORK. (Page 156).

Ex. 2. 14 f. 8 in. = 14·66 = height
62 6 = 62·5 = length
7333
29333
879999
9)916·666
101·8518
5 half bricks thick.
3)509·259
169·753 = content in yards.

(Page 157).

Ex. 3. 24·75 = length

17·5 = height

12375

17325

2475

2)433·125

216·5625

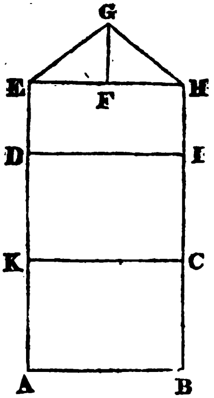
4

3)866·25

9)288·75

32·08⅓ yards.

Ex. 4. In this question, the wall may be considered as divided into three smaller walls, and one triangular gable, having for their common length 28 ft. 10 in. and their several heights as in the question; viz. AK=20, KD =20, DE=15 ft. 8 in., and GF 42 courses of bricks, which, at four courses to a foot, are equal to 10 ft. 6 in. Therefore, the contents of these being severally found, the sum will be the content of the whole wall.



ft. in.
First, 28 10 = AB
20 = AK

576 8
5 half bricks thick

3)2883 4

961 1 4 content of ACKB.

ft. in.
Secondly, 28 10 = CK
20 = KD

576 8
4 half bricks thick

3)2306 8

768 10 8 content of CDIK.

ft. in.
Thirdly, 28 10 = DI
15 8 = DE

432 6
19 2 8

451 8 8 content of DEHI.

	ft.	in.	
Lastly,	28	10	= EH
	10	6	= FG
	288	4	
	14	5	
3)	302	9	
	100	11	= content of EGHFE.

Then,	961	1	4
	768	10	8
	451	8	8
	100	11	
9)	2282	7	8

The content 253 5 7 8 or, in decimals, 253·62 yds.

Or, the work may be considerably abbreviated by the following operation:

ft.		h. bricks.		
20	×	5	=	100
20	×	4	=	80
$15\frac{2}{3}$	×	3	=	47
$5\frac{1}{4}$	×	2	=	$10\frac{1}{2}$
				$237\frac{1}{2}$

$$\frac{237\frac{1}{2}\times 28\frac{5}{6}}{3\times 9}=\frac{79\frac{1}{6}\times 28\frac{5}{6}}{9}=79\tfrac{1}{6}\times 3\tfrac{11}{54}=\frac{82175}{324}=253\tfrac{203}{324};$$

And, $253\frac{203}{324} = 253{\cdot}62$ yds. as before.

MASONS' WORK. (Page 158).

Ex. 2.

$$\begin{array}{r l}
24{\cdot}25 & = \text{length} \\
10{\cdot}75 & = \text{height} \\
\hline
12125 & \\
16975 & \\
24250 & \\
\hline
260{\cdot}6875 & \\
2 & = \text{thickness} \\
\hline
521{\cdot}375 & \text{the content.} \\
\hline
\end{array}$$

Ex. 3.

$$\begin{array}{r r r l}
 & 5 & 7 & = \text{length} \\
 & 1 & 10 & = \text{breadth} \\
\hline
 & 5 & 7 & \\
 & 4 & 7 & 10 \\
\hline
10 & & 2 & 10 = \text{content.} \\
\hline
\end{array}$$

Then,

$$\begin{array}{|c|c|r r}
\text{f.} & & s. & \\
2 & \frac{1}{6} & 8 & \\
 & & 10 & \\
\cline{3-4}
 & & 80 & \\
8'' & \frac{1}{3} & 1 & 4 \\
2'' & \frac{1}{4} & & 5\frac{1}{4} \\
 & & & 1\frac{1}{4} \\
\cline{3-4}
\multicolumn{2}{r}{20)} & 81 & 10\frac{1}{2} \\
\cline{3-4}
\end{array}$$

The value of the slab. $4 \quad 1 \quad 10\frac{1}{2}$

Ex. 4.

f.	in.	
4	6	
3	2	
13	6	
	9	
14	3	= content of mantle and slab.
4	4	
1	9	
4	4	
3	3	
7	7	content of the jambs.
14	3	
7	7	
21	10	the whole content.

CARPENTERS' AND JOINERS' WORK.

(Page 161).

Ex. 2. 36·25 = length
16·5 = breadth

18125
21750
3625

1/00)5 98·125

5 sq. 98 feet, ·125 or $\frac{1}{8}$ the answer.

Ex. 3.
173·8333 = length
10·5833 = height

5214999
52149999
1390666666
8691666666
173833333333

100)18397·3031663

18·3973 squares.

Ex. 4.
f. in.
52 8
3

2)158 0

79 0 = length
30 6 = breadth

100)2409 6

24·09 = content.

| 10*s*. | $\frac{1}{2}$ | 24·09 |
| 6*d*. | $\frac{1}{20}$ | 12·045
·60225 |

12·64725
20

12·945
12

11·34
4

1·36 Ans. 12*l*. 12*s*. 11¼*d*.

	f.	in.	
Ex. 5.	83	8	= compass
	12	6	= height
	1004		
	41	10	
	1045	10	content of the whole room, reckoning the door and shutters as work only.
Then,	7	8	
	3	6	
	23		
	3	10	
	26	10	
		3	
	80	6	content of three shutters.
	3	6	
	7		
	24	6	content of door
	80	6	
2)	105		
	52	6	= door and shutters as half work
	1045	10	
9)	1098	4	
	122	0	4 content in yards.

	in.		£.	s.	d.	
Then,	4	$\frac{1}{27}$		6		
				122		
				732		
					2½	
		20		732	2½	
			36	12	2½	answer.

SLATERS' AND TILERS' WORK.

(Page 163).

Ex. 2. The roof being of a true pitch, the girt over both sides is equal to the breadth of the building, plus its half: that is,

				ft.		£.	s.	d.	
	2)27	5							
	13	8	6	10	$\frac{1}{10}$	1	5	6	
								19	
The eaves on	41	1	6						
both sides =	2	8	0			24	4	6	
				5	½		2	6½	
	43	9	6	4	$\frac{1}{25}$		1	3¾	
length =	43	10		6′	⅛		1	0¼	
								1½	
	1883	0	6						
	36	5	11			24	9	5½	Ans.
	100)1919	6	5						
Content	19 sq. 19½ f.								

PLASTERERS' WORK. (Page 164).

Ex. 2.

f. in.
length 21 8
breadth 14 10

303 4
18 0 8

9)321 4 8

35 6 4 content in yards

| ft. 6 | $\frac{2}{3}$ |
| in. 4 | $\frac{1}{18}$ |

10

350
$6\frac{1}{2}$
$\frac{1}{4}$

12)$356\frac{3}{4}$

20)29 $8\frac{3}{4}$

£. 1 9 $8\frac{3}{4}$ the answer.

Ex 3.

ft. in.
18 6
12 3

222 0
4 7 6

9)226 7 6

yds. 25 1 7

| f. 1 | $\frac{1}{9}$ |

d.
8
25

200
$\frac{3}{4}$

12)$200\frac{3}{4}$

16 $8\frac{3}{4}$ value of the ceil.

Then,

```
 f.  in.                          f.  in.
 18  6                            12  3
 10  6                            10  6
 ------                           ------
185  0                           122  6
  9  3                             6  1  6
 ------                           ---------
194  3                           128  7  6
     2                                   2
 ------                           ---------
388  6 content of the 2 }        257  3  content of the 2 }
------ sides of room.   }        ------  ends of room.    }

          f.   in.        f.  in.        f.
         388   6          3   8          5
         257   3          7              5
         -------          ------         --
   From  635   9          25  8          25
   Take   50   8          25
         -------          ------
       9)595   1          50  8 deductions for door,
         -------          ------              &c.
   yds.   66   1   1 content of rendering.
          ---------

   | f. |     |    d.
   | 1  | 1/9 |    3
   |    |     |   66
                 ----
                 198
                   ¼
                 ----
             12)198¼
                ---------
                 16  6¼ value of rendering
                 16  8¾      do.   ceiling
                ---------
             £.1 13  3 whole amount.
                ---------
```

Ex. 4.

f.	in.		
14	5		= length
	10		deducted for cornice on both sides.
13	7		

f.	in.		
13	2		= breadth
	10		deducted for cornice as before
12	4		
13	7		
160	4		
7	2	4	
9)167	6	4	
18 yds. 5	6		of ceiling

Again,

f.	in.		
14	5		= length
9	3		= height
129	9		
3	7	3	
133	4	3	
		2	
266	8	6	content of two sides of the room.

f. in.
13 2 = breadth
9 3 = height

118 6
3 3 6

121 9 6
2

243 7 content of two ends of the room
266 8 6

510 3 6
28 0 0 deducted for door, 4 feet by 7

9)482 3 6 of rendering in feet.

53 yds. 5 3 of rendering.

f. in.
Lastly, 13 2
2

26 4 = double the breadth
28 10 = do. length

55 2
8 6 girt of cornice

36 9 4
2 3 7

39 yds. 0 11 of cornice.

PAINTERS' WORK. (Page 165).

	f.	in.		f.	in.	
Ex. 2.	10	4		4	4	
	20			4		
	206	8		17	4	fire-place.
		2				
From	413	4				
Take	17	4				
	396		content of two sides.			

	f.	in.		f.	in.	
	14	6		3	2	
	10	4		6		
	145			19		
	4	10		2		
	149	10		38		two windows.
		2				
From	299	8				
Take	38					
	261	8	content of two ends.			

	f.	in.		
Then,	261	8		
	396			
	9)67	8		
	73	0	8	or $73\frac{8}{108}$, or $73\frac{2}{27}$ yds.

```
              f.  in.                f.  in.
Ex. 3.        24   6                  3   6
              12   9                  7
             --------               --------
             294                     24   6 door
              18   4   6            --------
            -----------
             312   4   6
                       2
            -----------
             624   9       content of two sides.
            -----------

              f.  in.                f.  in.
              16   3                  8   6
              12   9                  1   3
             --------               --------
             195                      8   6
              12   2   3              2   1   6
            -----------             -----------
             207   2   3             10   7   6
                       2                      2
content of two -----------          -----------
ends  - - =  414   4   6             21   3 window breaks.
             624   9                -----------
            -----------
            1039   1   6       7   9
            -----------        3   6
   54   3                     ------
   21   3                     23   3
   24   6                      3  10   6
  -------                     -----------
  100   0 total of windows,   27   1   6
  -------             &c.              2
                              -----------
                              54   3     window shutters.
                              -----------
```

1039 1 6
100
1139 1 6
27 6 = deduction for fire-place
9)1111 7 6
123 yds. 4 7 6 total content.

5 6
5
27 6 fire-place

Then, for the cost,

yds.		123
3	⅓	6 pence per yard
		738
1	⅓	2 ½
	12	740½

20)61 8½
The answer £. 3 1 8½

GLAZIERS' WORK. (Page 166).

Ex. 2. 12 6
6 9
75 0
9 4 6
2)84 4 6
42 2 3 content

Then,

		42
2	⅙	10
		420 1¾ nearly
	12	421¾

20)35 1¾
The answer £. 1 15 1¾

	f.	in.			
Ex. 4.	7	9			
	6	6			
	5	3	3		
	19	6	3		
			3		
	58	6	9		
	1	10	5		
	60	5	2		= sum of heights
	3	9			= common breadth
	181	3	6		
	45	3	10	6	
	226	7	4	6	= total content.

	£.	s.	d.	
Then, 6 in. = ½		1	1	
			226	
	12	4	10	
1 = ⅕			6½	
4″ = ⅓			1	
			½	nearly
	12	5	6	the answer.

	f.	in.	
Ex. 3.	7	10	
	6	8	
	5	4	
	19	10	= sum of heights.

f.	in.	
3	11	= common breadth
	3	tiers.
11	9	

f.	in.	
19	10	
11	9	
218	2	
14	10	6
233	0	6 total of three tiers.

	£.	s.	d.
Then, 6′ = $\frac{1}{24}$		1	2
			233
	13	11	10
			$\frac{1}{2}$
	18	11	10$\frac{1}{2}$ the answer.

PAVERS' WORK. (Page 167).

					f.		£.	s.	d.	
Ex. 2.	27	10			3	$\frac{1}{3}$		3	2	
	14	9							45	
	389	8					7	2	6	
	20	10	6		1	$\frac{1}{3}$		1	0$\frac{1}{3}$	
					1	$\frac{1}{3}$			4	
9)	410	6	6		6	$\frac{1}{2}$			4	
					6	$\frac{1}{12}$			2	
	45	5	6	6					$\frac{1}{4}$	nearly
							7	4	4$\frac{1}{4}$	answer.

(Page 168).

Ex. 3.

63
45

2835 = content of the whole court-yd.

f.	in.
5	3
	63
9)330	9

36 6 9 = content of foot-path.

2835
330 9

9)2504 3

278 2 3 = content of part to be paved with pebbles.

		£.	s.	d.
6	⅔		3	
			36	
			108	
in. 9	⅛		2	
				3
		20)110		3
Price of foot-path		5	10	3

Again,

		£.	s.	d.
1	$\frac{1}{9}$		2	6
				278
		34	15	0
1	$\frac{1}{9}$			3⅓
3	¼			3⅓
				⅚
		34	15	7½

£.	s.	d.	
5	10	3	
34	15	7½	
40	5	10½	the ans.

PLUMBERS' WORK. (Page 169).

Ex. 2. 43
32

1376 = content of roof in feet
9·831 = lbs.

1376
4128
11008
12384

13527·456
840·522

14367·978 lbs.

57
2

114 = guttering

7·373 lbs.
114 feet

29492
7373
7373

840·522 lbs.

Then, 112)14367·978(128·285 cwt.
316
927
319
957
618

128·285
·9 = 18s. of a £.

115·4565
20

9·1300
12

1·5600
4

2·2400 Ans. £. 115 9 1½.

VAULTED AND ARCHED ROOFS.

PROBLEM I. (Page 170).

To find the Surface of a Vaulted Roof.

Ex. 2.

42·4
10·6

2544
4240

9)4494·4 feet

499·37 yds.

PROBLEM II. (Page 171).

To find the Content of the Concavity of a Vaulted Roof.

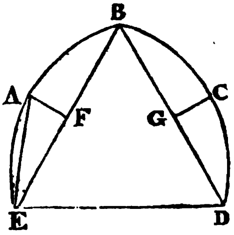
Ex. 2. First, 30 × 12 = 360

·7854
360

471240
23562

282·744 = area of the end
60 = length

Answer 16964·64 feet.



Ex. 3. Let the annexed figure represent the superficial end of the vault, which is evidently composed of the two segs. EABE and BCDB, and the equilateral triangle EBDE; the area of each of these must therefore be separately found, and the sum of those areas will be equal to the whole area of the end of the vault, which, being multiplied by the length, will give the content.

First, for segment EABE: the diameter of the circle of which this segment is a part is equal to $AE^2 \div AF$. (Euc. B. VI. Pr. XIII.) Thus, $EF = 25$, and $25^2 = 625$, $AF = 10$, and $10^2 = 100$, $625 + 100 = 725 = AE^2$, and $AE^2 \div AF = 725 \div 10 = 72{\cdot}5 =$ the diameter of the circle. Then, by problem 12, rule 3, of Mensuration of Superficies,

```
72·5)10·00(·137 675/725, or ·137 27/29
      2750
       5750
        675
```

The exact corresponding area to the number ·137$\frac{27}{29}$ will be found by the rule at page 322 of the Treatise.

To ·137 answers - - - -	·064760
The next area is - - - -	·065449
The diff. is - - - - -	·000689
$\frac{27}{29}$ of which is - - - -	·000641
Which, added to - - - -	·064760
Gives the number corresponding to ·137$\frac{27}{29}$	·065401

$$\begin{array}{rl}
\text{Then,}\quad \cdot065401 & \\
5256\cdot25 & = \text{sq. of diameter} \\
\hline
525625 & \\
21025000 & \\
2628125 & \\
3153750 & \\
\hline
343\cdot76400625 & = \text{area of one segment} \\
2 & \\
\hline
687\cdot5280125 & = \text{area of both segs.} \\
\hline
\end{array}$$

The area of the equilateral triangle is to be found by problem 7, rule 2, of Mensuration of Superficies. Thus,

$$\begin{array}{rl}
\cdot4330127 & = \text{tabular multiplier} \\
2500 & = \text{sq. of side} \\
\hline
1082\cdot53175 & = \text{area of triangle} \\
687\cdot5280125 & = \text{area of segments} \\
\hline
1770\cdot0597625 & = \text{whole area of the end} \\
20 & = \text{length} \\
\hline
35401\cdot19525 & \quad \text{the answer.} \\
\hline
\end{array}$$

PROBLEM III. (Page 171).

To find the Superficies of a Dome.

Ex. 2. The base being a regular hexagon, its area will be found by problem 7, rule 2, of Mensuration of Superficies. Thus,

2·598076 = tabular multiplier
100 = sq. of side

259·8076 = area of base
2

519·7152 the answer.

(Page 172).

Ex. 3. The area of the base is to be found by prob. 10, rule 3, of Mensuration of Superficies. Thus,

·07958
10000 = sq. of circumference

795·8 = area of base
2

1591·6 = double area of base.

The radius of the base is to be found by problem 8, rule 3, of Mensuration of Superficies. Thus,

3·1416)100·0000(2)31·8309 = diameter
57520
261040 15·9154 = radius.
97120
287200
4456

Then, by the rule,

As 15·9154 : 20 :: 1591·6
20

15·9154)31832·0(2000
12000

Therefore, 2000 feet = superficies required.

PROBLEM IV. (Page 172).

To find the solid Content of a Dome.

Ex. 2. The base being circular, its area will be found by problem 10, rule 2, of Mensuration of Superficies. Thus,

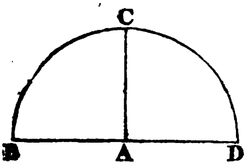
·7854
900 = sq. of diameter

706·86 = area of base
Height AC 15

10612·9
2

3)21225·8

7068·6 the answer.



PROBLEM VII. (Page 175).

To find the Concave Superficies of a Groin.

Ex. 2. 20
16

7)320 = area of base
$45\frac{5}{7}$

Answer $365\frac{5}{7}$

PROBLEM VIII. (Page 176).

To find the solid Content of a Groin Arch.

Ex. 2.
24
24
576 = area of base
8 = height
10)4608
460·8
4147·2 or $4147\frac{1}{5}$ the answer.

CONIC SECTIONS AND THEIR SOLIDS.

PROBLEM II.

In an Ellipse, to find the Transverse, or Conjugate, or Ordinate, or Absciss.

CASE I. (Page 233).

Ex. 2. First, 80 − 16 = 64 = greater absciss,
And 64
16
1024(32 mean proportional between 64 and 16
9
62)124
124
. . .

Then, as $8\cancel{0} : 6\cancel{0} :: 32$
6
8)192
24 the ordinate.

CASE II. (Page 234).

Ex. 2. 30 = one-half conjugate
30
900 24 = ordinate
576 24
324(18 576
1
28)224
224
...

As $6\cancel{0} : 8\cancel{0} :: 18$
8
6)144
24 = one-half diff. of abscisses
40 = one-half transverse
Take 64 = greater absciss
From 80 = transverse
16 = less absciss

Answer, 16 and 64.

CASE III. (Page 235).

Ex. 2. First, 70 — 14 = 56 = greater absciss,

```
    56
    14
   ----
   224
   56
   ----
   784(28 mean proportional between 56 and 14
   4
   ----          Then, as 28 : 20 : : 70
48)384                             70
   384                           ----
   ----                      28)1400
   ....                          ----
   ----                            50 answer.
                                 ----
```

CASE IV. (Page 236).

Ex. 2.

```
    20
     20
   ----
    400 = sq. of one-half conjugate
    256 = sq. of ordinate
   ----
    144(12
    1
   ----
 22)44
    44
    --
    ..
    --
```

The less absciss being given, this square root is to be *added* to one-half the conjugate,

12 + 20 = 32.

As 256 : 1440 : : 32
32

2880
4320

36 46080
40

1440 = rectangle of absciss
and conjugate 256)46080(180
2048

The transverse = 180.

PROBLEM III. (Page 237).

To find the Circumference of an Ellipse.

Ex. 2. 24
20

44

1·5708
44

62832
62832

69·1152 the answer.

PROBLEM IV. (Page 237).

To find the Area of an Ellipse.

Ex. 2.
·7854
24

31416
15708

18·8496
18

339·2928 the answer.

PROBLEM V. (Page 238).

To find the Area of an Elliptic Segment.

Ex. 2. 35)10·0(·285$\frac{5}{7}$ the corresponding segment (found by the rule laid down at p. 322 of the Treatise) is
300
200
25
$\frac{25}{35} = \frac{5}{7}$

·185166
25

4·629150
35

23145750
13887450

162·02025 the answer.

Ex. 3. 25)5·

·2 its corresponding segment is ·111823

25

2·795575

35

13977875

8386725

Answer 97·845125

PROBLEM VII. (Page 239).

To find any Parabolic Absciss, or Ordinate.

Ex. 2. 10

10

As 8 : 100 : : 18

18

8)1800

225

Then, $\sqrt{225} = 15$, the ordinate required.

Ex. 3. 18 12

18 12

As 324 : 18 :: 144

18

324)2592(8 the answer.

PROBLEM VIII. (Page 240).

To find the Length of a Parabolic Curve.

Ex. 2. 64 = sq. of ordinate   3 = absciss
12   3
76   9
4
Then $\sqrt{76} = 8{\cdot}71779$ 3)36
2
17·43558 the ans.   12

PROBLEM IX. (Page 241).

To find the Area of a Parabola.

Ex. 2. The length of the base is equal to the double ordinate. Therefore, 16
10
160
2
3)320
106$\frac{2}{3}$ the answer.

PROBLEM X. (Page 242).

To find the Area of a Parabolic Frustrum.

Ex. 2. 6 10 sq. of one end = 100
6 10 do. other = 36
36 100 diff. of squares = 64
6 10 3
216 1000 192
216

Diff. of cubes = 784
Double altitude = 6

192)4704(24½ the answer.
864
96

$\frac{96}{192} = \frac{1}{2}$

PROBLEM XII.

In an Hyperbola, to find the Transverse, or Conjugate, or Ordinate, or Absciss.

CASE I. (Page 243).

Ex. 2. First, 20 + 60 = 80, greater absciss,

And 80
20

1600(40 mean proportional between 80 and 20
16

. .

Then,

As 60 : 36 : : 40
40

60)1440

24 the ordinate.

CASE II. (Page 244).

Ex. 2. 18 = one-half conjugate 24 = ordinate
18 24

324 576
576

$\sqrt{900} = 30$

As 36 : 60 : : 30
30

36)1800

50 = one-half sum of abs.
30 = one-half trans.

80 = greater abs.
60 = transverse

20 = less abscis

Ans. 80 and 20.

CASE III. (Page 245).

Ex. 2. First, 20 + 60 = 80 = greater absciss
Then, 20
80

$\sqrt{1600} = 40$ = mean between 20 and 80.

As 40 : 24 :: 60
60

40)1440

36 = the conjugate.

CASE IV. (Page 246).

Ex. 2. 18 = ½ conjugate 24
18 24

324 576 = sq. of ordinate
576

$\sqrt{900} = 30$ Then, the less absciss being used,
Conjugate 36
30 + 18 = 48 Absciss 20

As 576 : 720 :: 48 720
48

576)34560(60 the transverse.

PROBLEM XIII. (Page 248).

To find the Length of an Hyperbolic Curve.

Ex. 2. First, to find an absciss by prob. 12, case 2,
30 10
30 10

900 = sq. of ½ conj. 100
100 = do. ordinate

$\sqrt{1000} = 31{\cdot}62.$

Then,

As $6\cancel{0} : 8\cancel{0} :: 31{\cdot}62$

```
                                  8
                             ------
                           6)252·96
                             ------
One-half sum of abscisses     42·16
               Transverse     40
                             ------
                  Absciss      2·16
                             ------
```

Then, by the rule,

```
      60            80           6400
      60            80             19
    ----          ----        -------
    3600          6400         121600
      21             9          75600
   -----         -----        -------
    3600         57600         197200
   7200          -----           2·16 = absciss
   -----                      -------
   75600                      1183200
   57600                      197200
   -----                     394400
  133200                     ---------
    2·16 = absciss           425952·00
  ------                     ---------
  799200
 133200
266400
---------
287712·00
---------
```

60
60

3600
80

288000
15

4320000 = 15 times trans. × square conj.

287712 425952
4320000 4320000

4607712 : 4745952 : : 20 = the double ord.
20

4607712)94919040(20·600
27648000
172800

Answer 20·600.

PROBLEM XIV. (Page 250).

To find the Area of an Hyperbola.

Ex. 2. 25
5

7)125

17·857142
50

67·857142
25

339285710
135714284

1696·42855

```
          1696·428550(41·187 = first root.
          16
          --
   81      96                      41·187
           81                          21
           --                      ------
  821      1542                     41187
            821                    82374
           ----                    ------
  8228      72185                  864·927
            65824                  -------
           ------
  82367      636150
             576569
            -------
              59581
         25
          50
        ----
        1250(35·355 = second root
        9
        --
65       350                       35·355
         325                            4
        ----                       ------
703       2500                     141·42
          2109                     864·927
          ----                    --------
7065       39100                  1006·347
           35325                       3000
          ------                 -----------
70705       377500                3019041·000
            353525                -----------
           -------
 30          23975
 25         ------      75
---                      50
750                     ----
  4                     3750 = 75 times trans.
----                    ----
3000
----
```

Then, 3750)3019041(805·074
19041
29100
28500
6000

The answer = 805·074: that in the Treatise being evidently incorrect.

PROBLEM XV. (Page 251).

To find the Solidity of a Spheroid.

Ex. 2.	50	·5236
	50	75000
	2500	26180000
	30	36652
	75000	Ans. 39270

Ex. 3.	7	·5236
	7	441
	49	5236
	9	20944
	441	20944
		230·9076 Answer.

PROBLEM XVI.

To find the Solidity of a Segment of a Spheroid.

CASE II. (Page 252).

Ex. 2.
10 = fixed axe
3
30
2 = double height
28
1 = square of height
28 As 100 : 36 :: 14·6608
·5236 36
41888 879648
10472 439824
14·6608 The answer 5·277888

Ex. 3.
30 = fixed axe
3
90
12 = double height
78
36 = square of height
468 As 900 : 2500 :: 1470·2688
234 2500
2808 900)3675672
·5236 Answer 4084·08
16848
8424
5616
14040
1470·2688

CASE II. (Page 253).

Ex. 2. 50
3

150 $\quad$ As $5\cancel{0} : 3\cancel{0} :: 3500$
Double height 10 $\quad$ 3

140 $\quad$ 5)10500
Squ. of height 25

$\quad$ 2100

700
280 $\quad$ Then ·5236
$\quad$ 2100

3500 $\quad$ 523600
$\quad$ 10472

$\quad$ 1099·56 Answer.

PROBLEM XVII.

To find the Content of the middle Frustrum of a Spheroid.

CASE I. (Page 254).

Ex. 2. 50 $\quad$ 40
50 $\quad$ 40

2500 $\quad$ 1600 $\quad$ 118800
2 $\quad$ ·2618

5000 $\quad$ 950400
1600 $\quad$ 118800
$\quad$ 712800
6600 $\quad$ 237600
18 = length

The ans. 31101·84

118800

CASE II. (Page 256).

Ex. 2. $$\left.\begin{array}{l}50 = \text{transverse}\\ 30 = \text{conjugate}\end{array}\right\}\text{ of the mid. section}$$

$$\left.\begin{array}{r}30\\ 18\end{array}\right\} = \text{The axes of each end}$$

$$\begin{array}{r}30\\ 18\\ \hline 540\\ \hline\end{array}$$

$$\begin{array}{r}1500\\ 2\\ \hline 3000\\ 540\\ \hline 3540\\ 40\\ \hline 141600\\ \cdot 2618\\ \hline 1132800\\ 141600\\ 849600\\ 283200\\ \hline 37070\cdot 8800\\ \hline\end{array}$$

37070·8800 the answer.

PROBLEM XVIII. (Page 258).

To find the Solidity of an Elliptic Spindle.

Ex. 2. 1. For the central distance, and axes of the ellipse.

```
  6          4·74773          6
  6          4·74773          3
  —          ———————          —
 36          1424319         18
  3         3323411           —
 ——        3323411          4·74773
108       1899092                 4
 ——      3323411            ———————
        1899092       From  18·99092
        —————————————  Take  18
        22·5409401529       ———————
                    4        ·99092
        —————————————       ———————
Take    90·1637606116
From    108
        —————————————
  99092)17·8362393884(18 nearly
             792703        or 4·5 = central distance
  one-half of greater diam.   3   = less absciss
                             ——
As 6* : 10 : : 15            7·5
               10              2
              ———            ———
           6)150              15  = conjugate
              ———              3  = less abs.
Trans. =       25              —
               —              12  = great abs.
        Half the length =     10  = ordinate
        15)3·0(·2 its tab. segment = ·111823
```

---

* 6 = mean proportional between the abscisses of conj. 12 and 3, found thus: 12 × 3 = 36, and $\sqrt{36} = 6$. (See Note at page 234 of the Treatise).

·111823
Transverse = 25

559115
223646

2·795575
Conjugate = 15

Area of seg. 41·933625
3

Length = 20)125·800875

6·29004375
Greater diam. 6

·29004375
Quadruple the cent. distance = 18

Take 5·2207875
From 36

30·7792125
Length = 20

615·5842500

Then, 615·58425
·5236

369350550
184675275
123116850
307792125

The answer 322·319913300 = 322·32

PROBLEM XIX. (Page 261).

To find the Solidity of a Frustrum or Segment of an Elliptic Spindle.

Ex. 2.

8
8

64 = square of greatest diameter
144 = square of double the middle diameter

208
10 = length

2080
·1309

18720
62400
2080

272·272 the answer.

Ex. 3.

10   6   17 = double mid. diam.
10   6   17

100   36   289
36
100

425
12 = length or height

5100
·1309

130900
6545

667·59 the answer.

```
Ex. 4.    16      12      29 = double mid. diam.
          16      12      29
         ----    ----    ----
         256     144     261
                 841     58
                 256     ----
                 ----    841
                1241
                   20 = length
                ------
                24820
                 ·1309
                ------
               223380
              744600
             24820
             ---------
             3248·9380 = answer.
             ---------
```

PROBLEM XX. (Page 262).

To find the Solidity of a Parabolic Conoid.

```
Ex. 2.          24          24192
                24           ·3927
                --         -------
                96          169344
               48           48384
              ----         217728
               576         72576
                42         ---------
              ----         9500·1984 answer.
              1152         ---------
             2304
             -----
             24192
             -----
```

PROBLEM XXI. (Page 262).

To find the Solidity of the Frustrum of a Paraboloid.

```
Ex. 2.          30          24
                30          24
               ----        ----
               900         576
               576
               ----
              1476
                 9 = height
             -----
             13284
             ·3927
             -----
             92988
            26568
           119656
           39852
          ---------
          5216·6268 answer.
          ---------
```

PROBLEM XXII. (Page 263).

To find the Solidity of a Parabolic Spindle.

```
Ex. 2.         6 = mid. diam.
               6
              --
              36          ·41888
              18             648
             ----        -------
             648          335104
             ----        167552
                        251328
                        ---------
          The answer    271·43424
                        ---------
```

PROBLEM XXIII. (Page 264).

To find the Solidity of the Middle Frustrum of a Parabolic Spindle.

Ex. 2.

18	10	10
18	10	18
324	100	180
8	3	4
2592	300	720
300		
720		
3612		
18		
65016		

65016
·05236

390096
195048
130032
325080

The answer 3404·23776

OF GAUGING.

PROBLEM XII. (Page 272).

To find the Content of a Cask of the first Form.

```
Ex. 2.        12          16
              12          16
             ----        ----
             144         256
                           2
                         ----
                         512
                         144
                         ----
                         656
                          20 = length
                       ------
                       13120
                      ·0009¼
                       ------
                      118080
                        3280
                      ------
                      12·136  = ale gallons.
                      ------

                       13120
                       ·0011⅓
                       ------
                      144320
                        ·437
                      ------
                      14·869  = wine gallons.
                      ------
```

PROBLEM XIII. (Page 274).

To find the Content of a Cask of the second Form.

```
Ex. 2.   12      16      16
         12      16      12
        ----    ----     --
        144     256       4 = diff. of diameters
                  2       4
                ----     --
                512      16
                144      ·4
                ----    ---
                656     6·4
                  6·4   ---
                -----
                649·6
                   20
                ------
                12992
                 ·0009¼
                ------
               116928
                 3248
               ------
               12·0176 = ale gallons
               ------
                12992
                 ·0011⅓
               ------
               142912
                 4330
               ------
               14·7242 = wine gallons.
               ------
```

PROBLEM XIV. (Page 275).

To find the Content of a Cask of the third Form.

Ex. 2.

$$\begin{array}{rrrr}
16 & 12 & & \cdot0014 \\
16 & 12 & & 8000 \\
\cline{1-2}\cline{4-4}
256 & 144 & \text{ale} = & 11\cdot2000 \\
144 & & & \\
\cline{1-1}\cline{4-4}
400 & & & \cdot0017 \\
20 & & & 8000 \\
\cline{1-1}\cline{4-4}
8000 & & \text{wine} = & 13\cdot6000 \\
\cline{1-1}\cline{4-4}
\end{array}$$

PROBLEM XV. (Page 276).

To find the Content of a Cask of the fourth Form.

Ex. 2.

$$\begin{array}{rr}
16 & 16 \\
12 & 12 \\
\hline
4 & 28 \\
4 & 28 \\
\hline
16 & 224 \\
 & 56 \\
\cline{2-2}
 & 784 \\
 & 3 \\
\cline{2-2}
 & 2352 \\
 & 16 \\
\cline{2-2}
 & 2368 \\
\cline{2-2}
\end{array}$$

Then,

Then,

$$
\begin{array}{rr@{\qquad\qquad}rr}
 & 2368 & & \\
\text{length} = & 20 & & \\
\hline
 & 47360 & & 47360 \\
 & \cdot00023\frac{1}{5} & & \cdot00028\frac{1}{3} \\
\hline
 & 142080 & & 378880 \\
 & 94720 & & 94720 \\
 & 9472 & & 15786 \\
\hline
\text{ale} = & 10\cdot98752 & \text{wine} = & 13\cdot41866 \\
\hline
\end{array}
$$

PROBLEM XVI. (Page 278).

To find the Content of a Cask by four Dimensions.

Ex. 2.

$$
\begin{array}{rr@{\qquad}r@{\qquad}rr}
14\frac{3}{8} = & 14\cdot375 & 16 & & 12 \\
 & 2 & 16 & & 12 \\
\hline
 & 28\cdot75 & 256 & & 144 \\
 & 28\cdot75 & & & 256 \\
 & & & & 826\cdot5625 \\
\hline
 & 14375 & & & 1226\cdot5625 \\
 & 20125 & & \text{length} = & 20 \\
\cline{5-5}
 & 23000 & & & 24531\cdot25 \\
 & 5750 & & & \cdot0004\frac{2}{3} \\
\cline{2-2}\cline{5-5}
 & 826\cdot5625 & & & 9\cdot812500 \\
 & & & & 1\cdot635416 \\
\cline{5-5}
 & & & \text{ale} = & 11\cdot447916 \\
\end{array}
$$

24531·25
·0005$\frac{2}{3}$

12·265625
1·635416

wine = 13·901041

PROBLEM XVII. (Page 279).

To find the Content of any Cask from three Dimensions only.

Ex. 2.	16	12	16
	16	12	12
	256	144	192
	39	·25	26
	2304	720	1152
	768	288	384
	9984	3600	4992
		9984	
		4992	

18576
20 = length

371520
·00034

148608
111456

9)126·3168

14·0352 = wine.

And,

371520
·00003$\frac{1}{11}$

11·14560
·3377

11·4883 = ale.

OF THE ULLAGE OF CASKS.

PROBLEM XVIII. (Page 280).

To find the Ullage of a Cask.

Ex. 2. 16 = bung diam. 12 = head diam.
16 12

256 144
144 square of one-half length of cask = 100

112
25 = sq. of distance of the surface of the liquor from middle of cask

100)2800

3)28

9·333
256 = square of bung diameter

246·666
5

459·05)1233·330(3·43 = quantity of liquor contained between the middle of cask and surface of the ullage.
156183
125633
17915

Then,

Then, 12·136 = content of whole cask

6·068 = one-half ditto
3·43

2·638 = ullage required.

(Page 281).

Ex. 3. 32)8

·25 whose tab. seg. is ·153546
92

307092
1381914

4)14·126232
3·531558

Answer 17·657790

SPECIFIC GRAVITY.

PROBLEM I. (Page 285).

To find the Magnitude of any Body from its Weight.

oz.
Ex. 2. As 922 : 16 : : 1728
16

922)27648(30 oz. nearly.

Ounces.

Ex. 3. As 925 : 35840 : : 1

925)35840(38$\frac{138}{185}$ the answer.

8090

$$5\left|\frac{690}{925}\right| = \frac{138}{185}$$

PROBLEM II. (Page 286).

To find the Weight of a Body from its Magnitude.

Ex. 2. As 1728 : 35·25* : : 922

922

7050
7050
31725

1728)32500·50(19 oz. nearly
15220

Ex. 3.

10
3

30
$2\frac{1}{2}$

60
15

75 = solid content of the block.

Then, as 1 : 75 : : 925
75

4625
6475

16)69375

$4335\frac{15}{16}$ = the answer.

---

* 1 pint, ale measure, = 35¼ inches.

THE WEIGHT AND DIMENSIONS OF BALLS AND SHELLS.

PROBLEM I. (Page 290).

To find the Weight of an Iron Ball.

Ex. 2.

5·54
5·54

2216
2770
2770

30·6916
5·54

1227664
1534580
1534580

170·031464

8)170·031464

8)21·253933
2·656741

23·910674 or 24 lbs. nearly.

PROBLEM II. (Page 291).

To find the Weight of a Leaden Ball.

Ex. 2.

5·24
5·24

2096
1048
2620

27·4576
5·24

1098304
549152
1372880

143·877824

143·877824
2

9)287·755648

31·972849 or
32 lbs. nearly.

PROBLEM III. (Page 292).

To find the Diameter of an Iron Ball from its Weight.

Ex. 2. 24
$7\frac{1}{9}$

168
2·666

170·666. Take 5, as being the nearest cube to this; the cube of which is 125. Then, by the second rule for the cube root at page 41 of the Treatise, proceed thus:

125 $170\frac{2}{3}$
2 2

250 $340\frac{2}{3}$
$170\frac{2}{3}$ 125

As $420\frac{2}{3}$: $465\frac{2}{3}$: : 5 or,
3 3

As 1261 : 1397 : : 5
5

1261)6985(5·54 nearly.
6800
4950

PROBLEM IV. (Page 293).

To find the Diameter of a Leaden Ball from its Weight.

Ex. 2. 8
9

2)72

36

The nearest cube is 3, the cube of which is 27.

Then 27 36
2 2
54 72
36 27
As 90 : 99 :: 3
3
90)297(3·3
270

Again, for a second operation, taking 3·3, its cube is 35·937

Then 35·937 36
2 2
71·874 72
36 35·937
As 107·874 : 107·937 : 3·3
3·3
323811
323811
356·1921

107·874)356·1921(3·301927 the answer.
325701
207900
1000260
293940
781920
26802

PROBLEM V. (Page 294).

To find the Weight of an Iron Shell.

Ex. 2.

9·8
9·8

784
882

9604
9·8

76832
86436

941·192

7
7

49
7

343

941·192
343

598·192
9

64)5383·728(84·120
263
77
132
40

Answer 84⅛ lbs. nearly.

PROBLEM VI. (Page 295).

To find how much Powder will fill a Shell.

Ex. 2.

7
7

49
7

57·3)343·0(5·986 or 6 lbs. nearly.
5650
4930
3460
22

PROBLEM VII. (Page 296).

To find how much Powder will fill a Rectangular Box.

$$\begin{array}{lr}
\text{Ex. 2.} & 12 \\
 & \underline{12} \\
 & 144 \\
 & \underline{12} \\
 & 30)\underline{1728} \\
 & 57\tfrac{18}{30} = 57\tfrac{3}{5}\ \text{lbs.}
\end{array}$$

PROBLEM VIII. (Page 297).

To find how much Powder will fill a Cylinder.

$$\begin{array}{lr}
\text{Ex. 2.} & 4 \\
 & \underline{4} \\
 & 16 \\
 & \underline{12} \\
 & 38{\cdot}2)192{\cdot}0(5\tfrac{10}{382}\ \text{or}\ 5\tfrac{5}{191}\ \text{lbs.} \\
 & \underline{10} \\
 & 382
\end{array}$$

PROBLEM IX. (Page 298).

To find the Size of a Shell to contain a given Weight of Powder.

$$\begin{array}{lr}
\text{Ex. 2.} & 57{\cdot}3 \\
 & \underline{6} \\
 & 343{\cdot}8
\end{array}$$

The cube root of which is 7, the num. of lbs. reqd.

PROBLEM X. (Page 299).

To find the Size of a Cubical Box to contain a given Weight of Powder.

Ex. 2.

$$\begin{array}{r} 400 \\ 30 \\ \hline 12000 \end{array}$$

The nearest cube root is 22, the cube of which is 10648

Then,

$$\begin{array}{rcrcr} 10648 & & 12000 & & \\ 2 & & 2 & & \\ \cline{1-1}\cline{3-3} 21296 & & 24000 & & \\ 12000 & & 10648 & & \\ \cline{1-1}\cline{3-3} 33296 & : & 34648 & :: & 22 \\ & & 22 & & \\ \cline{3-3} & & 69296 & & \\ & & 69296 & & \\ \cline{3-3} \end{array}$$

$$\begin{array}{l} 33296)762256(22 \cdot 89 \text{ inches.} \\ \qquad\quad 96336 \\ \qquad\quad 297440 \\ \qquad\quad\;\; 310720 \\ \qquad\quad\;\;\; 11056 \end{array}$$

PROBLEM XI. (Page 300).

To find what Length of a Cylinder will be filled by a given Weight of Gunpowder.

Ex. 2. 38·2
20

64)764·0(11
124
60

$\overline{64} = \frac{15}{16}$

Answer $11\frac{15}{16}$.

THE PILING OF BALLS AND SHELLS.

PROBLEM I. (Page 301).

To find the Number of Balls in a Triangular Pile.

Ex. 2. 21
20

420
22

840
840

6)9240

1540 the answer.

PROBLEM II. (Page 302).

To find the Number of Balls in a Square Pile.

Ex. 2.

```
     21
      20
    ----
    420
     41
    ----
    420
   1680
  ------
 6)17220
  ------
    2870 the answer.
  ------
```

PROBLEM III. (Page 303).

To find the Number of Balls in a Rectangular Pile.

Ex. 2.

```
     59
      3
    ----
    177
     19
    ----
    158
     20
    ----
   3160
     21
   -----
   3160
  6320
  ------
 6)66360
  ------
  11060 the answer.
  ------
```

PROBLEM IV. (Page 304).

To find the Number of Balls in an incomplete Pile.

Ex. 2. 24
25

120
48

600
26

15600 = six times number in the complete pile.

7
8

56
9

504 = six times num. in the pile wanting at top.

Then, 15600
504

6)15096 = six times the whole

2516 the answer.

Ex. 3.

24	7
25	8
120	56
48	15
600	280
49	56
29400	840
840	
6)28560	
4760 the number in the incomplete pile.	

Ex. 4. First, 40 − 12 = 28 = length } of the upper
20 − 12 = 8 = breadth } pile.

Then, by prob. 3,

40	28
3	3
120	84
19	7
101	77
20	8
2020	616
21	9
6)42420	6)5544
7070	924
924	
6146 the answer.	

OF DISTANCES, AS ASCERTAINED BY THE VELOCITY OF SOUND.

(Page 306).

Ex. 3.

7
3
14)21
$1\frac{1}{2}$ mile.

Ex. 4. First, 6 pulsations = $\frac{6}{70}$ or $\frac{3}{35}$ of a minute.

Therefore, 60 seconds
3
35)180(5·14285 = time in seconds
50
150
100
300
200
25

Then, 5·14285
3
14)15·42855
1·10203
1760
612180
71421
10203
179·57280

Therefore, 1 mile 179 yds. = distance required.

Ex. 5. First, 5 pulsations = $\frac{5}{75}$ or $\frac{1}{15}$ of a minute = 4 seconds.

Then,
$$\begin{array}{r} 4 \\ 3 \\ \hline 12 \\ \hline 14 \end{array} = \tfrac{6}{7} \text{ of } 1760 \text{ yds.}$$

$$\begin{array}{r} 1760 \\ 6 \\ \hline 7)10560 \\ \hline 1508\tfrac{4}{7} \\ \hline \end{array} = \text{yds.}$$

MISCELLANEOUS QUESTIONS. (Page 307).

Ex. 1.
$$\begin{array}{rr} 28 & \qquad 14 \\ 20 & \qquad 10 \\ \hline 560 & \qquad 140 \\ 280 & \qquad 2 \\ \hline \text{Difference} = 280 & \qquad 280 \\ \hline \end{array}$$

$$\begin{array}{r} 560 \\ 280 \\ \hline 100)840 \\ \hline \end{array}$$

$8\frac{4}{10}$ or $8\frac{2}{5}$ squares

$$\begin{array}{r} 45s. \\ 8\tfrac{2}{5} \\ \hline 360 \\ \tfrac{2}{5} \text{ of } 45 = \quad 18 \\ \hline 2'0)37'8 \\ \hline \text{Amount} = 18 \quad 18 \\ \hline \end{array}$$

Ex. 2. Since the area of a rectangle is found by multiplying its length into its breadth, it follows that the area being divided by the length will give the breadth. Therefore,

f. in.
144 14 3 = 171 inches
9
171)1296 = inches in one square yard
$7\frac{99}{171}$ the answer.

Ex. 3.
yds. ft.
144 6
9
28)1032
$36\frac{24}{28}$ or $36\frac{6}{7}$ the answer.

Ex. 4.
$2\frac{1}{2}$
7
$17\frac{1}{2}$
2
3)35
$11\frac{2}{3}$ the answer.

Ex. 5. First, 102
21
2142
2
4284 = superficial content of the 2 sides

3*s.* 2*d.* = 38*d.*, and 38 ÷ 6 = $6\frac{1}{3}$ yds. in the whole.

Then,

Then, 144
9

1296 = inches in one square yard
$6\frac{1}{3}$

7776
432 = $\frac{1}{3}$ of 1296

From 8208 = inches in the whole trough
Take 4284

3924 = inches in the bottom and 2 ends: and these being considered as forming one continued plank, the width will be found, as in Ex. 2 and 3, by dividing by the length, which is equal to 102 + 21 + 21 = 144,

144)3924(27¼ inches, the answer.
1044
$\frac{36}{144} = \frac{1}{4}$

Ex. 6. 47 9   47 9
47 9   4

2243 3   191 0 = content of foot-path
35 10 7

2280 1 7 = content of the whole yard
191

9)2089 1 7 = content of yard, exclusive of foot-path

232 yds. 1 f. 1 in.

Then,

Then, at 6*d*. per yd. ½ £ ½ in. 7′ = about ½.

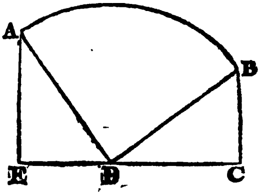
And, 6*d*.
232

12)1392½

20)11′6½

5 16 0½ the answer.

Ex. 7 *. First, in the triangle AED we have the perp. and hyp. to find the base: and in the triangle BDC, we have the same.



First, in the triangle AED,

40	33
40	33
1600	99
	99

Take 1089
From 1600

511(22·605 = DE
4

42)111
84

446)2700
2676

45205)240000
226025
13975

---

* Construction, similar to that in Prob. 3, Ex. 6, Mens. of Sup.

Secondly, in the triangle BDC,

40 21
40 21

From 1600 441
Take 441

1159(34·04
9

64)259
256

6804)30000
27216

2784

22·605
34·04

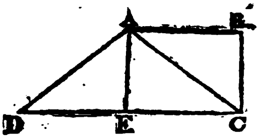
56·645
12

7·74

Ans. 56 f. 7·75 in. nearly.

Ex. 8. 100*l.* = 24000 pence
18)24000

1333·333



which is equal to the area of the triangle ACDA, or of the parellelogram ABCEA. Then, as the perpendicular from the vertex of an isosceles triangle divides the base or third side into two equal parts, we shall have CE or AB = one-half of DC = 44: Therefore,

44)1333·333(30·3 = perpendicular AE or BC
133
13

And, in the triangle ACE we have base (CE) and perpendicular (AE), to find the hyp. (AC),

30·3	44
30·3	44
909	176
9090	176
918·09	1936
	918·09

2854·09(53·42 = side AD or AC
25

103)354
309

1064)4509
4256

10682)25300
21364

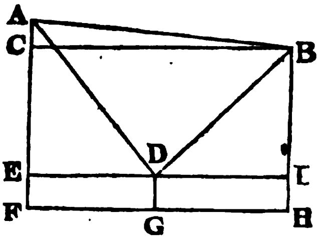
3936

Then, 53·42
2

106·84 Answer.

Ex. 9. Let AF and BH be the two columns, and DG the statue. Then, by the assistance of the imaginary lines EI and CB, we have GH equal to DI, and BI to CE: also AC, the difference between the height of the columns = 14 feet.

First, in the triangle BDI, we have the base (DI) and hypothenuse (DB), to find the perp. (BI),



76
76

$5776 = \text{DI}^2$

86
86

$7396 = \text{DB}^2$
5776

$\sqrt{1620} = 40{\cdot}249 = \text{BI or CE.}$

Again, 40·249 + 14 = 54·249 = AE   97 = AD
54·249   97

$\text{Take } 2942{\cdot}954001 = \text{AE}^2$ 9409
$\text{From } 9409 = \text{AD}^2$

$\sqrt{6466{\cdot}045999} = 80{\cdot}41 = \text{ED}$
76 = DI

The distance between the 2 cols. = 156·41 = EI or CB.

Lastly, 156·41 = CB
156·41

$24464{\cdot}0881 = \text{CB}^2$
$196 = \text{AC}^2$

$\sqrt{24660{\cdot}0881}$ = 157·03 feet = AB, the distance between the tops of the two columns.

Ex. 10. First, 16·5 ÷ 2 = 8·25 = circumference.
Then, as 22 : 7 :: 8·25
7

22)57·75

2·62 feet, = the diam.

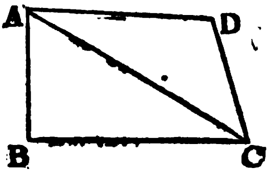
Ex. 11.
5
4
20

3·1416 = tab. number
20
62·8320 = 63 feet nearly.

Ex. 12. First, 21*s.* = 252*d.*
·4330127 = tabular number
8
3·4641016

Then, 3·4641)252·000(72·746 feet, the answer.
95230
259480
15993

Ex. 13. The area of triangle ABC (it being right-angled), is thus found,



31·2
13
2)405·6
202·8 = area of ABC

Then, for side AC,
13
13
169

31·2
31·2
624
312
936
973·44
169
$\sqrt{1142{\cdot}44} = 33{\cdot}6$ = side AC

And

And for the area of the triangle ADCA we have the 3 sides: (Mens. of Sup. prob. 2, rule 2).

```
         18
         24
         33·6
        ─────
       2)75·6
        ─────
         37·8          37·8          37·8
         18            24            33·6
        ─────         ─────         ─────
         19·8          13·8           4·2
          4·2
        ─────
          396
         792
        ─────
         83·16         43379·5824(208·2
          13·8         4
        ─────         ──────
         66528        408)3379
        24348              3264
        8316              ──────
      ─────────      4162)11558
        1147·608            8324
           37·8             ─────
      ─────────             3234
         9180864            ─────
        8033256
       3442824
      ──────────
      43379·5824
      ──────────
```

208·2 area of ADCA
202·8 area of ABCA

411· area of the whole.

Ex. 14.

$$\begin{array}{rr} 24 & 8 \\ 14 & 6 \\ \hline 345 & 4 \\ 12 & 4 \\ \hline 357 & 8 \\ & 8 \text{ lbs.} \\ \hline 112)2861 & 4 \\ \hline \text{cwt. } 25 & 2 \quad 4 \\ \hline \end{array}
\qquad
\begin{array}{|l|c|rrr} \text{qu.} & & & s. & \\ 2 & \frac{1}{2} & & 18 & \\ & & & 25 & \\ \cline{3-4} \text{lbs.} & & 22 & 10 & \\ 4 & \frac{1}{14} & & 9 & \\ & & & & 7\frac{3}{4} \\ \cline{3-5} \multicolumn{2}{r}{\pounds.} & 22 & 19 & 7\frac{3}{4} \text{ Ans.} \\ \cline{3-5} \end{array}$$

Ex. 15.

First, 27)144(5·3333 = *ab*
90
90
9

$$\begin{array}{r} 58 \\ 5{\cdot}3333 \\ \hline 52{\cdot}666 \\ \hline \end{array} = bx,$$

52·666)144·000(2·736 = *bn*
386680
17198

$$\begin{array}{r} 27 \\ 2{\cdot}736 \\ \hline 24{\cdot}265 \\ \hline \end{array} \text{ nearly} = on$$

24·265)144·000(5·93 = *nc*
226750
73650

$$\begin{array}{r} 52{\cdot}666 \\ 5{\cdot}93 \\ \hline 46{\cdot}736 \\ \hline \end{array} = cw$$

46·736)144·000(3·08 = *cm*
379200
5312

$$\begin{array}{r} 24{\cdot}265 \\ 3{\cdot}08 \\ \hline 21{\cdot}185 \\ \hline \end{array} = pm$$

21·185)144·000)6·79 = md
168900
20605

46·736
6·79
39·946 = dv

39·946)144·000(3·604 = dl
241620
1944

21·185
3·604
17·581 = ql

17·581)144·000(8·13 = le
33520
15939

39·946
8·13
31·816 = eu

31·816)144·000(4·52 = ke
167360
8280

17·581
4·52
13·061 = ki

13·061)144·000(11·02 = kf
13490
329

31·816
11·02
20·796 = ft

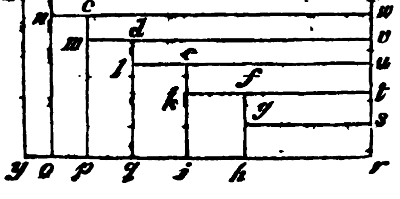
20·796)144·000(6·92 = fh
19224

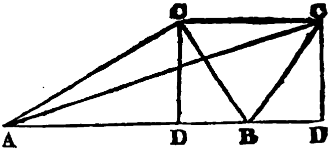
13·061
6·92
6·141 = fh

Therefore the dimensions of the remaining piece will be 20·796 by 6·141. The small difference between this answer and that in the Treatise, arises probably from the continual rejection of the 3rd place of decimals in the quotient.

The following solution to this question appeared in the Ladies' Diary for the year 1823: and though the nature of the operation is rather different from the generality in this work, the author has taken the opportunity of inserting it here, as it furnishes a general solution to questions of this kind, and as its result in this instance will be found to coincide very nearly with that in the Treatise.

Suppose a = area of the whole slab = 10⅞ square feet, and b = its breadth = 2¼ feet. Then will the areas of the first, second, third, &c. remaining pieces be denoted by $a - 1$; $a - 2$, $a - 3$, &c. respectively; also, the slab being rectangular, the area of each piece divided by its breadth or length, will give the other dimension; and since the first foot is cut from the length, the breadth of the first remainder will be $= b$; the length of the first and second $= (a - 1) \div b] = \frac{a-1}{b}$; breadth of second and third $= a - 2 \div \frac{a-1}{b} = \frac{b\,.\,(a-2)}{a-1}$; length of third and fourth $= \frac{(a-1)\,.\,(a-3)}{b\,.\,(a-2)}$; breadth of fourth and fifth $= \frac{b\,.\,(a-2)\,.\,(a-4)}{(a-1)\,.\,(a-3)}$, &c.; hence it is manifest, that the length of every even remainder is = the product of the areas of all the preceding odd ones, divided by the breadth of the slab into the product of the areas of the like even ones; and the breadth = its area divided by the length: therefore the length of the last remainder will be $= \frac{(a-1)\,.\,(a-3)\,.\,(a-5)\,.\,(a-7)\,.\,(a-9)}{b\,.\,(a-2)\,.\,(a-4)\,.\,(a-6)\,.\,(a-8)} = \frac{79\,.\,7\,.\,47\,.\,31}{71.22.13.23}$ feet = 20·70206 inches; and the breadth = ⅞ × 144 ÷ 20·70206 = 6·08635 inches, the dimensions as required.





Ex. 16. First, 1 acre = 160 perches. 160 ÷ $20(\frac{AB}{2})$ = 8 = perp. CD, and $\sqrt{(CB^2 - CD^2)} = \sqrt{(20^2 - 8^2)} = \sqrt{336}$ = 18·330302 = DB; hence AD = AB ± BD = 40 + 18·330302 or 40 − 18·330302, the root of every positive quantity being either + or −.

Again, $AC = \sqrt{(AD^2 + DC^2)} = \sqrt{(58{\cdot}330302^2 + 8^2)} =$ 58·87634686. Or, $AC = \sqrt{(21{\cdot}669698^2 + 8^2)} = 23{\cdot}09925922$. Either of these results, therefore, may be the side required.

Ex. 17. First, $\frac{1}{3}$ of 40 feet = 13 f. 4 in.

```
   24  6          326  8
   13  4               2 half bricks
  -------        --------
  318  6       3)653  4
    8  2         --------
  -------         217  9    = content of upper
  326  8         ----------                part
       4 half bricks
  -------
3)1306  8
 ----------
   435  6  8 = content of lower part
```

33 course of bricks at 4 course to a foot = 9 f. 2 in.

```
      24  6
       9  2
     ------
     220  6
       4  1            435  6  8 = first 1/3
     ------            326  8    = second 1/3
   3)224  7            217  9  4 = third 1/3
     ------             37  5  2 = gable
   2)74 10  4         -----------
     --------  272)1017  5  2 = content of the
Gable = 37  5  2      -----------           whole.
       --------          3  201  5  2
                        -------------
```

£.	s.	
5	10	
	3	
16	10	
4	1	$3\frac{1}{4}$ = value of 201 feet
		$2\frac{1}{2}$ = do. 5 in. 2″
20	11	$5\frac{3}{4}$ the answer.

Ex. 18. First, $12 \times \frac{16}{2} = 96$ = area of the larger triangle.

As $96 : 256 = (16^2) :: 24$

24

1024

512

96)6144(64, then $\sqrt{64} = 8$ the perp. of the less triangle.

384

Again, as $96 : 144 = (12^2) :: 24$

24

576

288

96)3456(36, then $\sqrt{36} = 6$, base of less triangle.

576

Lastly, $8^2 = 64$, and $6^2 = 36$, $64 + 36 = 100$,

$\sqrt{100} = 10$, hypothenuse of less triangle;

therefore, the 3 sides are 6, 8, and 10.

Ex. 19. By problem 7 of Conic Sections, proceed thus:

840
612
1680
840
5040
514080

514080
·7854
2056320
2570400
4112640
3598560
4,*03758·4320
4
·15032
40
6·01280

Therefore, 4 ac. or 6 p. = the quantity of ground inclosed by the rails.

Again, 1 link = 7·92 in. and 14 inches = 1·7676 links. Therefore, the measure of the square the longer way including the rails = $840 + \overline{2 \times 1{\cdot}7676}$: and the shorter way = $612 + \overline{2 \times 1{\cdot}7676}$: or 843·5352 and 615·5352. Then proceed as before: thus,

843·5352
615·5352
16870704
42176760
25306056
42176760
42176760
8435352
50612112
519225·60803904 which being multiplied by ·7854 = 407799·7925232, the area of the greater ellipse, including the rails.

4·077997925
4·037584320
·040413605 = diff. of 2 ellipses
4840 = yds. in one acre
195·6018482
9 = feet in 1 sq. yd.
1760·4166338 or 1760½ feet = the quantity of ground occupied by the walls.

---

* 100000 links = 1 acre of land.

Ex. 20. As 4 : 49 = (7²) :: 40
40

4)1960

490

Then, √490 = 22·135 the answer.

Ex. 21. First, ½ acre = 2240 yds.
·7854)2420·0(3081·23 = sq. of diam. of circle
63800
9680 √3081·23 = 55·5 = diam.
18260
25520 2)55·5
1958
27·75 the answer.

Ex. 22. 22 9
3

4)68 3

17 0 9 = rafters
2

34 1 6
1 8 = eaves on both sides

35 9 6 = whole breadth
32 9

1145 4
26 10 1

100)1172 2 1

11 72 2 = content in squares.

15s.
11

8 5 feet
10 9½ = 72 &c.

£. 8 15 9½ answer.

Ex. 23. By the question, a *fathom* 9 inches in compass would weigh 44 lbs.

$$\text{As } 9^2 : 44 :: 12^2 : \frac{44 \times 144}{81} =$$

$$\frac{44 \times 9 \times 16}{9 \times 9} = \frac{44 \times 16}{9} = \frac{704}{9}$$

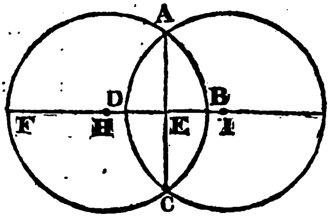
$$\frac{704}{9} = 78\tfrac{2}{9} \text{ the answer.}$$

```
Ex. 24.  74      26      74      16
         46      46      26      26
        ----    ----    ----    ----
         444     156     444     416
        296     104     148        3
        ----    ----    ----    ----
        3404    1196    1924    1248 = 3 stays
           2       2    ----    ----
        ----    ----
        6808    2392
        2392    ----
        1924
        1248
      ------
  144)12372 = whole content in inches
      ------
```

$85\frac{11}{12}$ = do. in feet; which, at 28 lbs. each, give 85 qu. 25 lbs., or 22 cwt. 1 qu. 25 lbs.

Then,

```
1 qu.  ¼  22                                 469
          21s.                                12
         ----                               ----
          462                           7)5628
            5  6                            ----
25 lbs. =   5                               804 = content of
         ------                              12   workshop
          472  6                            ----
            3  6    22f. 10in.=274)9648=do. in inches
         ------                             ----
          469=expense of                   35 f. 5 in. the ans.
          ---   paving workshop.
```



Ex. 25. First, to find the height DE or EB of the segment AECD; HB = 25 by the question; and HI = 30; therefore, HE = 15, and HB less HE = 25 less 15 = 10 = DE or EB = height of the segment.

Then, by the rule, Mens. of Sup. prob. 12, rule 3.

50)10

·2 to which corresponds in the table ·111823
sq. of diameter 2500

55941500
223646

area of the one segment = 279·5875
2

area of ABCD as required 559·175

cwt.
Ex. 26. As 20 : 10 : : 50
5

2)50

25 cwt. = weight of 50 ft. if 1¼ in.

Again,

Again, $1\frac{1}{4} = \frac{5}{4} = \frac{10}{8}$, and $(\frac{10}{8})^2 = \frac{100}{64}$, also $(\frac{7}{8})^2 = \frac{49}{64}$.

cwt.

As $\frac{100}{64}$: 25 :: $\frac{49}{64}$,

Or, as 100 : 25 :: 49

$$\begin{array}{r} 49 \\ \hline 225 \\ 100 \\ \hline 100)1225 \\ \hline 12{\cdot}25 \end{array}$$

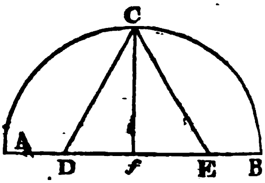
12·25 cwt. = 12 cwt. 1 qu.

1 cwt. or 112 lbs. at $3\frac{1}{2}d.$ per lb. = £. 1 12 8.

$$\begin{array}{l|l|rrr} \text{qu.} & & & & \\ 1 & \frac{1}{4} & 1 & 12 & 8 \\ & & & & 12 \\ \hline & & 19 & 12 & \\ & & & 8 & 2 \\ \hline & & \text{£. } 20 & 0 & 2 \end{array}$$

the answer.

Ex. 27. First (by rule 2, prob. 7, Mens. of Sup.)



Tab. num. = ·4330127)100·00000(230·9401

13397460

40707900

1736757

$\sqrt{230{\cdot}9401} = 15{\cdot}196$ = side of the triangle

2)15·196

7·598 = half side Df.

Here we have DC = 15·196 = (hyp.) and D*f* = 7·598 = (base) of the right-angled triangle DC*f*, to find C*f* (perp.): thus,

7·598 From 230·9401 = DC²

7·598 Take 57·729604 = D*f*²

60784 173·210496 = C*f*²

68382

37990 Then √173·210496 = 13·16 = C*f*, which is the radius of the semicircle.

53186

57·729604 Therefore 13·16 × 2 = 26·32 the diam.

Ex. 28. 1·25 1·2271875 = area of the bore or hollow of the pipe

1·25

1·5625 1·2271875

·7854 36 inches in 1 yard

62500 73631250

78125 36815625

125000

109375 44·17875 = solid content of the hollow within the pipe.

1·22718750

inches. ounces. inches.

As 1728 : 11325 :: 44·17875

11325

22089375

8835750

13253625

48596625

1728)500324·34375

Weight of the hollow within the pipe 289·5742

Ounces in 14 lbs. = 224

Weight of pipe, including the space within; that is, taking the whole as solid, } = 513·5742

Again,

Again, as 11325 : 1728 : : 513·5742

1728

11325)887456·2176

·78·3625 = solid content of pipe and vacuity in inches, taking (as before) the whole as being solid.

Then, inches in one yd. = 36)78·3625

2·1767 = area of the circular end of pipe.

·7854)2·1767(2·771454 = sq. of diam. of do.

60590

56120

11420

35660

42440

3170

2·771454(1·66476

1

26 177

156

326 2114

1956

3324 15854

13296

33287 255800

233009

22791

1·66476 = diam. of end of pipe.

1·25 = bore

2)·41476 = twice the thickness of the lead.

·20738 of an inch, the answer.

£.
Ex. 29. 10
20

200
12

28*d.*)2400(85$\frac{5}{7}$ = 85·71428 = area of semicircle
160
20
— = $\frac{5}{7}$
28

85·71428 × 2 = 171·42856 = area of the whole circle

·7854)171·42856(218·26 = square of diameter
14348
64945
21136
54280
7156

Then, $\dot{2}1\dot{8}\cdot2\dot{6}$(14·773 the answer.
1

24)118
96

287)2226
2009

2947)21700
20629

29543)107100
88629

18471

```
Ex. 30.  289          83521
         289           ·7854
        ─────        ───────
         2601         334084
        2312         477605
        578         668168
       ──────      584647
        83521      ─────────
       ──────   3)655197·3934 = area of circle
                  ─────────
                 218399·1311 = ⅓ or seg. to be cut off.
                  ─────────
```

Again, ·7854 = area of a circle, whose diameter is 1; and ⅓ of this = ·2618, to which corresponds in the table at the end of the Treatise ·3675,

```
                  ·3675
                    289
                 ──────
                  33075
                 29400
                 7350
                ───────
        Take    106·2075 = height of seg.
        From    144·5 ,  = radius of circle
                ───────
                 38·292  = distance between the
                *38·292    centre of circle and
                ───────    middle of the chord.
                  76584
                324628
                76584
              306366
             114886
             ───────────
             1466·177264
             ───────────
```

---

* The remaining part of the operation is proceeded with by rule 2, prob. 3, Mens. of Sup. The radius forming the hypothenuse, the distance (as above), the perp., and half the required line, the base of a right-angled triangle.

144·5
144·5

7225
5780
5780
1445

20880·25
1466·17726

$\surd$19414·072 = 139·3358 half the chord.

Therefore, 139·3358
2

278·6716 = the chord required.

Ex. 31.

64·3
12

771·6
12

8)9259·2 = sq. inches

1157·4
6$\frac{5}{9}$

6944·4
643 = $\frac{5}{9}$ of 1157·4

7587·4 = oz.
21

75874
151748

159335·4

34944 = ounces in 1 fother of lead

34944)159335·4 (4·5597
195594
208740
340200
257040
12432

20

11·1940
12

2·328

Answer £. 4 11 2

Ex. 32. Let x = diameter, and $a = 3{\cdot}1416$. Then ax = circumference: and $x \times ax = ax^2$, the superficial content of the sphere. Again, let $c = {\cdot}5236$, then, by the rule, $x^3 \times c = cx^3$ = solidity of the whole sphere. And, by the question, $ax^2 = cx^3$; but $\frac{ax^3}{6}$ = solidity also. Therefore, as things which are equal to the same are equal to each other, $ax^2 = \frac{a}{6}x^3$. This equation divided by x^2, gives $a = \frac{a}{6}x$, or $6a = ax$. This again divided by a, gives $x = 6$; which is the diameter required.

Note.—The answer to this question may be determined without the assistance of algebra, by dividing the tabular number 3·1416 by ·5236; these being the numbers respectively used for finding the superficies and solidity of the sphere. Thus,

·5236)3·1416(6 as before.

Ex. 33.

$$\begin{array}{r l}
22{\cdot}5 & \\
22{\cdot}5 & \\
\hline
506{\cdot}25 & \\
{\cdot}07958 & \\
\hline
40{\cdot}287375 & = \text{area of a circle, whose circumference is } 22{\cdot}5 \\
\hline
\end{array}$$

$$\begin{array}{r l}
45 & \\
45 & \\
\hline
2025 & \\
{\cdot}07958 & \\
\hline
161{\cdot}4950 & = \text{area of a circle, whose circum. is } 45 \\
\hline
\end{array}$$

Then,

As 40·287375 : 3 : : 161·495
3
40·287375)48·34485
Answer 12 bushels.

Ex. 34. 3·5
3·5
12·25
·7854
4900
6125
9800
8575
9·62115 = area of circle, whose diam. is 3½ feet
Then, 7·25 = 7¼ inches
2
12)14·5
1·208 = double breadth of the curb in feet
3·5
4·708 = diam. of cir. including curb
4·708
22·165264 = sq. of diam.
·7854
From 17·4085983456 = area of circle
Take 9·62115
7·78 = area of curb, or difference between the two circles
8
12)62·24
5·186 = 5*s.* 2¼*d.* the answer.

Ex. 35 *.

$$\begin{array}{r} 100 \\ 80 \\ \hline 2)8000 \\ \hline 4000 \\ \hline \end{array} = \tfrac{1}{2} \text{ area of the garden.}$$

Now, suppose x = breadth of the walk. Then will $100x$ = area of the walk parallel to the longer side: and $80 - x$ will equal the length of the shorter side; therefore $(80 - x) \times x = 80x - x^2$ = the area of the walk parallel to the shorter side. And, by the question,

$$100x + 80x + x^2 = 4000 = \tfrac{1}{2} \text{ the area of garden}$$

$$\text{Or, } 180x - x^2 = 4000$$

$$\text{That is, } x^2 - 180x = -4000$$

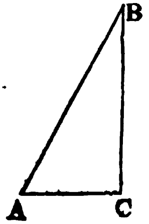
By completing the square, $x^2 - 180x + 8100 = 4100$

By Evolution, $x \pm 90 = \sqrt{4100} = \pm 64{\cdot}031$

$$x = 90 - 64{\cdot}031 = 25{\cdot}969$$

Therefore, 25·969 is the breadth required.

Ex. 36. By the figure, it is plainly seen that the piece broken forms the hypothenuse, and the remaining part the perpendicular of a right-angled triangle: the distance between the foot of the may-pole and the foot of the broken piece being the base.



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* This question is incorrectly inserted in the Treatise, and should be thus: " A gravel walk is to be made down *one* side, and across *one* end," &c.

Therefore,

$$\begin{array}{r} 15 \\ 15 \\ \hline 225 = \mathrm{AC}^2 \end{array} \qquad \begin{array}{rr} & 39 \\ & 39 \\ \hline & 1521 = \mathrm{AB}^2 \\ & 225 \\ \hline & 1296(36 = \mathrm{BC} \\ & 9 \\ \hline 66 & 396 \\ & 396 \\ \hline & \cdots\cdots \\ \hline \end{array}$$

$\mathrm{BC} + \mathrm{AB} = 36 + 39 = 75$ the whole length.

Ex. 37.

$$\begin{array}{rl} & 60 \\ & 60 \\ \hline 7) & 3600 = \text{sq. of diameter} \\ \hline & 514{\cdot}555(22{\cdot}68 = \text{7th man's share} \\ & 4 \\ \hline 42) & 114 \\ & 84 \\ \hline 446) & 3055 \\ & 2676 \\ \hline 4528) & 37950 \\ & 36224 \\ \hline & 1726 \\ \hline \end{array} \qquad \begin{array}{rl} & 514{\cdot}555 \\ & 2 \\ \hline & 1029{\cdot}11(32{\cdot}08 \\ & 9 \\ \hline 62) & 129 \\ & 124 \\ \hline 6208) & 51100 \\ & 49664 \\ \hline & 1436 \\ \hline \end{array}$$

$$\begin{array}{r} 32{\cdot}08 \\ 22{\cdot}68 \\ \hline 9{\cdot}4 = \text{6th man's share} \\ \hline \end{array}$$

```
      514·555
            3
     ---------
      1543·666(39·28
      9        32·08
      --       -----
    69)643     7·20 = 5th man's share
       621     -----
       ---
   782)2266            514·555
       1564                  4
       ----            --------
        702           2058·222(45·36
        ---            16      39·28
                       --      -----
                     85)458     6·08 = 4th man's share
                        425
                        ---
                   903)3322
                       2709
                       ----
                        613
      514·555           ---
            5
     ---------
     2572·777(50·71
     25       45·36
     ----     -----
 1007)7277    5·35 = 3rd man's share
      7049
      ----
      ·128               60
      ----               55·56
     514·555             -----
           6              4·44 = 1st man's share
     ---------            ----
     3087·333(55·56
     25       50·71
     --       -----
  105)587     4·85 = 2nd man's share
      525     -----
      ---
 1105)6233
      5525
      ----
       708
       ---
```

Ex. 38. First, 16 feet 6 in. = 16·5

16·5
16·5

825
990
165

272·25
3

81675

816·75 (28·57
4

48)416
384

565)3275
2825

450

·57
12

6·84

Answer 28 f. 7 in. nearly.

Or the new side $= \sqrt{(16·5^2 \times 3)} = 16·5 \sqrt{3} = 16·5 \times 1·73205 = 28·578825$, as before.

Ex. 39.

3
3

9
3

27

12
12

144
12

1728

27)1728(64
108

Or thus:
12 ÷ 3 = 4 and $4^3 = 64$.

Ex. 40. First, one acre = 4840 yds.

·7854)4840·0(6162·46
12760
49660
2536

6162·46(78·5 = diameter of circle
49

148)1262
1184

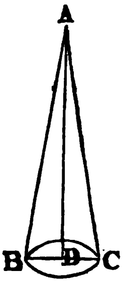
1565)7846
7825

21

2)78·5

39·25 = 39¼ yards.

Ex. 41. Let the annexed figure represent the cone: and first, 64 ÷ 3·1416 = 20·3717 = diameter of base BC, or 10·3717 = radius of base BD or DC. The radius of the base, with the *perp.* height of the cone, form the two legs of the triangle ABDA: therefore, $10{\cdot}3717^2 + 118^2 = 14031{\cdot}57216089 = AB^2$, and the square root of this = 118·45109, which is the *slant* height of the cone. Then, prob. 5 of Solids,



```
  118·45109
         64 = circum.
  ---------
  47380436
 71070654
 ----------
2)7580·86976
 ----------
9)3790·43488
 ----------
  421·15 or 421 yds. 1 ft.
```

```
Lastly,
  421
     8d. per yard
  ----
12)3368
  ----
  280   8
        ¾ = val. of 1 ft.
 ---------
20)280  8¾
 ---------
   14  0  8¾ the answer.
 ---------
```

```
Ex. 42.  18·5          As 8 : 342·25 :: 7·5
         18·5                    8
         ----                 ------
          925          7·5)2738·00(365
        1480                 488
        185                    380
        ------                   5
        342·25
        ------
```

Then √365 = 19·106 the diam. required: all superficial bodies being to each other as the squares of their similar parts.

Ex. 43. 3·1416
6 = diameter

18·8496 = circumference
6

113·0976 = convex surface in feet
144

16286·0544 = do. in inches
3*d.*

½ = ⅙ 48858
8143

12)57001

2,0)475,0 1

£.237 10 1 the answer.

Ex. 44. 4 1·5 1·5
4 1·5 4

16 2·25 6
16
6

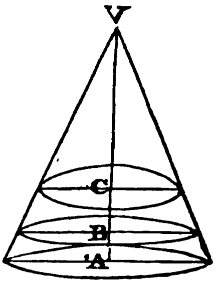
24·25 = sum of squares of diams. and prod.
·2618

6·348650 = mean area
7·9 = perp. height, found as in ex. 41

57137850
44440550

50·1543350
12*s.*

601·85202 = £.30 1 10¼ the answer.



Ex. 45. Similar cones being to each other as the cubes of their altitudes, it will be,

as $VA^3 : VB^3 :: 3 : 2$ or
$VA : VB :: \sqrt[3]{3} : \sqrt[3]{2}$

also,

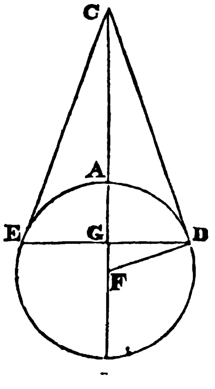
$VA^3 : VC^3 :: 3 : 1$ or
$VA : VC :: \sqrt[3]{3} : 1.$

But $VA = 20$, the whole height of the cone; therefore, we shall have from the first of these statements $VB = 20\sqrt[3]{\tfrac{2}{3}} = 17{\cdot}4716107$ = the height of the upper and middle parts together.

From the second statement, we deduce $VC = 20\sqrt[3]{\tfrac{1}{3}} = 13{\cdot}8672247$, the height of the upper part VC. Consequently, $17{\cdot}4716107 - 13{\cdot}8602247 = 3{\cdot}604386$, the height of the middle part BC. And $20 - 17{\cdot}4716107 = 2{\cdot}5283893$, the height of the lower part AB.

Ex. 46. First, $300 \times 200 = 60000$ = area of the garden,
$300 + 300 + 200 + 200 + 8 + 8 + 8 + 8 = 1032$ = feet round garden, including ditch,

$$\begin{array}{r} 1032 \\ 8 \\ \hline 8256 \end{array} = \text{area of ditch.} \qquad 8256)60000(7{\cdot}26 \text{ feet, the answer.}$$
$$\begin{array}{r} 22080 \\ 5568 \end{array}$$



Ex. 47. The surfaces of segments being as their altitudes, the altitude of the segment in the question must be one-third of the diameter: GF, therefore, $= \tfrac{1}{6}$ of the diam. or $\tfrac{1}{3}$ of FD. But, by similar triangles,

$FG : FD :: FD : FC.$

And as $GF = \tfrac{1}{3}$ of FD, $FD = \tfrac{1}{3}$ of FC, or $FC = 3FD = 3$ times radius.

Then, $3FD$ (or FC) $- AF = 2FD =$ diameter, which is the height to which the spectator must be raised.

Ex. 48. ·025

·025

·000625

·7854

2500

3125

5000

4375

·000490875 = area of circle, whose diam. is $\frac{1}{40}$.

Again, 1 cubic foot = 1728 inches,

·0000490875)1728·0000(3520244·5

25537500

9937500

12000

3520244·5 inches; which, if reduced, is 97784·6 yards, or 55 miles 984 yards.

Ex. 49. First, $4^3 = 64$, and $\frac{1}{10} = ·1$,

As 9 : 64 : : 24

24

256

128

9)1536

$\sqrt[3]{170·666} = 5·546$

·1

5·646 inches.

Ex. 50. First, $4^3 = 64$. Then,

$$\begin{array}{l} \text{As } 9 : 64 :: 1 \\ \qquad\quad 1 \\ \hline \quad 9)64 \\ \hline \end{array}$$

$$\left.\begin{array}{r} \sqrt[3]{7\cdot111111}=1\cdot923=\text{diam.} \\ \cdot0392=\tfrac{1}{49}\text{ of diam.} \\ \hline 1\cdot9622=\text{caliber gun} \\ \hline \end{array}\right\}\text{of a 1 lb. ball.}$$

$$\begin{array}{l} \text{As } 9 : 64 :: 2 \\ \qquad\quad 2 \\ \hline \quad 9)128 \\ \hline \end{array}$$

$$\left.\begin{array}{r} \sqrt[3]{14\cdot222222}=2\cdot4228=\text{diam.} \\ \cdot0495=\tfrac{1}{49}\text{ of diam.} \\ \hline 2\cdot4723=\text{caliber gun} \\ \hline \end{array}\right\}\text{2 lb. ball.}$$

$$\begin{array}{l} \text{As } 9 : 64 :: 3 \\ \qquad\quad 3 \\ \hline \quad 9)192 \\ \hline \end{array}$$

$$\left.\begin{array}{r} \sqrt[3]{21\cdot333}=2\cdot7734=\text{diam.} \\ \cdot0567=\tfrac{1}{49}\text{ of diam.} \\ \hline 2\cdot8301=\text{caliber gun} \\ \hline \end{array}\right\}\text{3 lb. ball.}$$

$$\begin{array}{l} \text{As } 9 : 64 :: 4 \\ \qquad\quad 4 \\ \hline \quad 9)256 \\ \hline \end{array}$$

$$\left.\begin{array}{r} \sqrt[3]{28\cdot444}=3\cdot0526=\text{diam.} \\ \cdot0623=\tfrac{1}{49}\text{ diam.} \\ \hline 3\cdot1149=\text{caliber gun} \\ \hline \end{array}\right\}\text{4 lb. ball.}$$

As 9 : 64 :: 6

$$\begin{array}{r} 6\\ \hline 9)384\\ \hline \end{array}$$

$$\sqrt[3]{42{\cdot}666} = \left.\begin{array}{rl} 3{\cdot}4943 = & \text{diam.}\\ {\cdot}0713 = & \tfrac{1}{49}\text{ diam.}\\ \hline 3{\cdot}5656 = & \text{caliber gun}\\ \hline \end{array}\right\}\text{6 lb. ball.}$$

As 9 : 64 :: 12

$$\begin{array}{r} 12\\ \hline 9)768\\ \hline \end{array} \qquad \left.\begin{array}{rl} 4 = & \text{diam.}\\ {\cdot}0816 = & \tfrac{1}{49}\text{ diam.}\\ \hline 4{\cdot}0816 = & \text{caliber gun}\\ \hline \end{array}\right\}\text{9 lb. ball.}$$

$$\sqrt[3]{85{\cdot}333} = \left.\begin{array}{rl} 4{\cdot}4026 = & \text{diam.}\\ {\cdot}0898 = & \tfrac{1}{49}\text{ diam.}\\ \hline 4{\cdot}4924 = & \text{caliber gun}\\ \hline \end{array}\right\}\text{12 lb. ball}$$

As 9 : 64 :: 18

$$\begin{array}{r} 18\\ \hline 9)1152\\ \hline \end{array}$$

$$\sqrt[3]{128} = \left.\begin{array}{rl} 5{\cdot}0397 = & \text{diam.}\\ {\cdot}1028 = & \tfrac{1}{49}\text{ diam.}\\ \hline 5{\cdot}1425 = & \text{caliber gun}\\ \hline \end{array}\right\}\text{18 lb. ball.}$$

As 9 : 64 :: 36

$$\begin{array}{r} 36\\ \hline 9)2304\\ \hline \end{array}$$

$$\sqrt[3]{256} = \left.\begin{array}{rl} 6{\cdot}3496 = & \text{diam.}\\ {\cdot}1296 = & \tfrac{1}{49}\text{ diam.}\\ \hline 6{\cdot}4792 = & \text{caliber gun}\\ \hline \end{array}\right\}\text{36 lb. ball.}$$

As 9 : 64 :: 24

$$\begin{array}{r} 24 \\ \hline 9)1536 \\ \hline \end{array}$$

$$\left.\begin{array}{rl} \sqrt[3]{170\cdot666} = 5\cdot5469 = & \text{diam.} \\ \cdot1132 = & \frac{1}{49}\text{ diam.} \\ \hline 5\cdot6601 = & \text{caliber gun} \\ \hline \end{array}\right\}\text{24 lb. ball.}$$

As 9 : 64 :: 42

$$\begin{array}{r} 42 \\ \hline 9)2688 \\ \hline \end{array}$$

$$\left.\begin{array}{rl} \sqrt[3]{298\cdot666} = 6\cdot6844 = & \text{diam.} \\ \cdot1364 = & \frac{1}{49}\text{ diam.} \\ \hline 6\cdot8208 = & \text{caliber gun} \\ \hline \end{array}\right\}\text{42 lb. ball.}$$

Ex. 51. First,

$$\begin{array}{r} 60)4\cdot6 \\ \cdot0766 \\ \hline 4\cdot523 \\ \hline \end{array} \qquad \begin{array}{r} 60)5\cdot8 \\ \cdot0966 \\ \hline 5\cdot703 \\ \hline \end{array} \qquad \begin{array}{r} 60)8\cdot0 \\ \cdot133 \\ \hline 7\cdot867 \\ \hline \end{array}$$

$$\begin{array}{r} 60)10\cdot \\ \cdot1666 \\ \hline 9\cdot833 \\ \hline \end{array} \qquad \begin{array}{r} 60)13\cdot \\ \cdot2166 \\ \hline 12\cdot783 \\ \hline \end{array} = \text{the several diameters of the shells.}$$

Then,

Then, by problem 5 of Balls and Shells,

4·6 × $\frac{7}{10}$, or ·7 = 3·22 the inner diameter,

3·22 × 3·22 × 3·22 = 33·386248

$$\begin{array}{rr} 4·523 \times 4·523 \times 4·523 = & 92·529403667 \\ & 33·386248 \\ \hline & 59·143155 \\ & 9 \\ \hline & 64)532·288395 \\ \hline \text{Wt. of shell for 4·6 mortar} = & 8·317006 \\ \hline \end{array}$$

5·8 × $\frac{7}{10}$, or ·7 = 4·06 the inner diameter,

4·06 × 4·06 × 4·06 = 66·923416

$$\begin{array}{rr} 5·703 \times 5·703 \times 5·703 = & 185·485563927 \\ & 66·923416 \\ \hline & 118·562147 \\ & 9 \\ \hline & 64)1067·059323 \\ \hline \text{Wt. of shell for 5·8 mortar} = & 16·672802 \\ \hline \end{array}$$

8 × $\frac{7}{10}$, or ·7 = 5·6 the inner diameter,

5·6 × 5·6 × 5·6 = 175·616

$$\begin{array}{rr} 7·867 \times 7·867 \times 7·867 = & 486·886183363 \\ & 175·616 \\ \hline & 311·270183363 \\ & 9 \\ \hline & 64)2801·431650267 \\ \hline \text{Wt. of shell for 8 mortar} = & 43·772369535 \\ \hline \end{array}$$

10 × $\frac{7}{10}$, or ·7 = 7 the inner diameter.

7 × 7 × 7 = 343

9·833 × 9·833 × 9·833 = 950·732022537

343

607·732022537

9

64)5469·588202833

Wt. of shell for 10 mortar = 85·462315669

13 × $\frac{7}{10}$, or ·7 = 9·1 the inner diameter,

9·1 × 9·1 × 9·1 = 753·571

12·783 × 12·783 × 12·783 = 2088·807192687

753·571

1335·236192687

9

64)12017·125734183

Weight of shell for 13 mortar = 187·767589596

Again, for the weight of powder in each shell by prob. 6 of Balls and Shells,

3·22 × 3·22 × 3·22 = 33·386248, which, divided by 57·3 = ·58265, weight of powder for 8·317 shell.

4·06 × 4·06 × 4·06 = 66·923416, which, divided by 57·3 = 1·16794 = weight of powder for 16·672 shell.

5·6 × 5·6 × 5·6 = 175·616, which, divided by 57·3 = 3·06481 = weight of powder for 43·772 shell.

7 × 7 × 7 = 343, which, divided by 57·3 = 5·986 = weight of powder for 85·462 shell.

9·1 × 9·1 × 9·1 = 763·571, which, divided by 57·3 = 13·1513 = weight of powder for 187·767 shell.

Lastly, for weight of shell filled,

8·317
·583
8·900 = wt. of first

16·672
1·168
17·840 = wt. of second

43·684
3·065
46·749 = wt. of third

85·462
5·986
91·448 = wt. of fourth

187·767
13·151
200·918 = wt. of last.

Ex. 52. By rule at page 301 of the Treatise,

59
30
1770
29
15930
3540
6)51330
.8555 the answer.

Ex. 53. By rule at page 302 of the Treatise,

49
3
147
18
129
19
2451

2451
20
6)49020
8170 the answer.

Ex. 54. By rule at page 301 of the Treatise,

52
51
52
260
2652

2652
50
6)132600
22100 the answer.

Ex. 55. By rule at page 303 of the Treatise,

52
51
52
260
2652
50
6)132600
22100 = number in the whole pile
1330 = do. in the part wanting at top
20770 the answer,

19
20
380
21
6)7980
1330

Ex. 56. First, the sides of the bottom course of the pile will be 40 + 11 and 10 + 11 = 51 and 21.

Then, by the rule at page 303 of the Treatise,

40	51
3	3
120	153
9	20
111	133
10	21
1110	133
11	266
6)12210	2793
	22
2035	
	6)61446

10241 = number in whole pile
2035 = number in pile wanting at the top

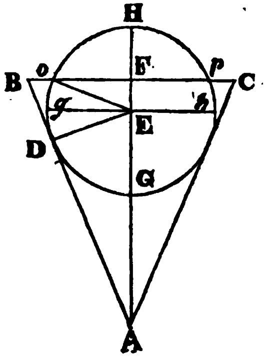
The answer 8206

Ex. 57. First,
36 = AF²
6·25 = BF² or CE²

42·25(6·5 = AB or AC
36

125)625
625

...



Then, by similar triangles,

As 2·5 : 6 :: 2
2
2·5)12
4·8 = AD

4·8
4·8
23·04
4
27·04(5·2 = AE
25
102)204
204
....

2
2
4

6 = AF
5·2 = AE
·8 = EF

2· = EH
·8 = EF
1·2 = FH

2
2
$4 = EO^2$
$\cdot 64 = EF^2$
$\sqrt{3 \cdot 36} = 1 \cdot 1833 =$ OF.

Then, by the assistance of these particulars, we shall readily find the solidity of the segment OPH; which being taken from the solidity of the whole sphere, will leave the solidity of the segment *op*G below the surface of the glass, and, consequently, the quantity of water which will run over. (Prob. 13, Mens. of Solids).

3·36 = sq. of radius of base
3

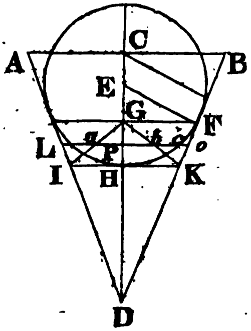
10·08
1·44 = sq. of height, or FH

11·52
1·2 = height FH

Take 13·824
From 64 = cube of diameter of sphere

50·176
·5236

26·2721536 the answer.



Ex. 58. Suppose the surface of the water to rise to LO, by the immersion of the sphere. Then, we have by the question BC = 2·5, CD = 6, EF = 2 given, to find PH, the height of the segment, immersed.

First, $\sqrt{CD^2 + BC^2} = BD$ = 6·5: and by the nature of similar triangles,

As BC : BD :: EF : DE
or, 2·5 : 6·5 :: 2 : 5·2.

Again, as CB : CD :: EF : DF
or, as 2·5 : 6 :: 2 : 4·8.

And, as DE : DF :: DF : DG
or, as 5·2 : 4·8 :: 4·8 : 4·43077. Then,

DE − EH = DH, or 5·2 − 2 = 3·2 : also, DG − DH = GH,
or 4·43077 − 3·2 = 1·23077 : and,

As DC : AB :: DH : IK
or, as 6 : 5 :: 3·2 : $2\frac{2}{3}$ or $\frac{8}{3}$ = IK.

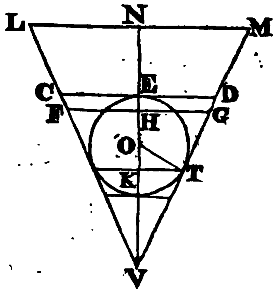
Now the small ungula, or space between the water and the cone, viz. *c*OF, where the sphere is immersed, is equal to the small cone G*ab*.

Then, $\text{IK}^2 \times \cdot 7854 \times \frac{1}{3}\text{GD} - \frac{1}{5}$ (the cone ABCD) = solidity of *c*OF entirely round the glass; viz. $(\frac{8}{3})^2 \times \cdot 7854 \times \frac{4 \cdot 43077}{3} = 8 \cdot 2487152782$. Also, the solidity of the cone ABCD $= \frac{5^2 \times \cdot 7854 \times 2}{5} = 7 \cdot 854$. Therefore, by subtraction, 8·2487152782 − 7·854 = ·394715278 = solidity *c*OF, or of the small cone G*ab*.

Again, $\text{IK}^2 \times \cdot 7854 \times \frac{\text{GH}}{3} = 2 \cdot 2913108$ = solidity of cone IGK. Then, by the similarity of solid bodies,

As IGK : G*ab* :: $\text{GH}^3 : \text{GP}^3$; that is,
As 2·2913 : ·394715 :: $(1 \cdot 23077)^3 : (\cdot 68477)^3$.

which is equal to GP^3. Therefore, GP = ·68477, and GH − GP = 1·123077 − ·68477 = ·546, which is equal to the height PH, or the part of the axis of the sphere which is immersed.



Ex. 59. Let LVM be the whole cone; FVG = $\frac{1}{5}$ of it: the solidity of which = 7·854 found by the last problem. Then, we have by the question VN = 6, LM = 5. And, by the nature of solid bodies, $\sqrt[3]{\frac{6^3}{5}}$ = VH = 3·508821. Therefore,

As VN : LM :: VH : FG, or
As 6 : 5 :: 3·508821 : 2·924.

And GH = ½FG = 1·462. Again,

As VH : HG :: KT : KO.

Here it will be necessary, in order to arrive at the true answer, to suppose KT = 1, and to continue the work of the problem with that assumed number.

Then, by the last proportion, KO will equal ·41666.

And, as KO : KT :: KT : KV,
Or, as ·4166 : 1 :: 1 : 2·4,

and, $VO = \underline{VK + KO} = 2\cdot81666$. Therefore,

$OT = \sqrt{KT^2 + KO^2} = 1\cdot083332$.

And VE = VO + OE or OT, = 3·9.

As VK : KT :: VE : ED, or
As 2·4 : 1 :: 3·9 : 1·625.

CD = 2ED = 1·625 × 2 = 3·25. Now, with these similar dimensions, we can obtain the solidity of the cone CDV: thus, $\frac{(3\cdot25^2)\times\cdot7854\times VE}{3} = 10\cdot78454$, also OT × 2 = 2·166664 = diameter of the globe; and, again, $2\cdot166664^3 \times \cdot5236 = 5\cdot325682$ = solidity of globe: therefore, by the water's rising when the sphere is immersed from FG to CD, the content of FCDG must be equal to the solidity of the globe, or 5·325682. Again, from the solid content of CDV take the last solid content of FCDG; viz. 10·78454 − 5·32568 = 5·45886 = solidity of FVG. But the real solidity of FVG was found by the last problem to be 7·854. Therefore, by making a proportion between the real and supposed solidity, we shall arrive at the true solidity of the cone CDV.

As 5·45886 : 7·854 :: 10·78454 : 15·5163,

which is the true content of the cone CDV; and 15·5163 − 7·854 = 7·6623 the true content of CDGF; which is, as was before proved, equal to the solidity of the globe: then, by reversing the rule for finding the solidity of a sphere, we get $\sqrt[3]{(7\cdot6623 \div \cdot5236)} = 2\cdot44599$ = the diameter as required.

Ex. 60.

33
3

14)99

$7\frac{1}{14}$ miles.

Ex. 61.

17
3

14)51

$3\frac{9}{14} = 3\frac{2}{3}$ nearly.

Ex. 62. By problem 3, Specific Gravity,

12
7

5 As 5 : 12 : : 1000 = spec. grav. of water.
12

5)12000

2400 = density of first

$14\frac{1}{2}$
9

$5\frac{1}{2}$ As $5\frac{1}{2}$: $14\frac{1}{2}$: : 1000
Or, as 11 : 29 : : 1000
29

11)29000

$2636\frac{4}{11}$ = density of second

Therefore, the proportion is as $2636\frac{4}{11}$ to 2400; or, as 145 to 132 nearly.

Ex. 63. First, $3\frac{1}{2}$ lbs. troy = 42 oz.
·48926
231 = No. of inches in a gallon of brandy

113·01906 = weight of brandy
42 = weight of bottle

155 01906 = weight of bottle and brandy

1·36)42·00

30·882 = No. of inches of glass

231
30·882

261·882 = No. of inches in bottle and brandy
·5427

142·123 = weight in salt water

155·0190
142·123

oz. 12·896 = the diff., and, consequently, the force necessary to prevent it from sinking.

Ex. 64. 1000 = specific gravity of water
50000

16)50000000 = weight in ounces

2240)3125000 = do. in lbs.

$1395\frac{1}{11}$ = answer in tons.

FINIS.

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